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THE OPTICS OF X-RAYS*

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In opening this symposium on the optics of the extremes of the spectrum, it may be well to review briefly the immense range of the electromagnetic radiations. In Fig. 1 these various rays are graphically

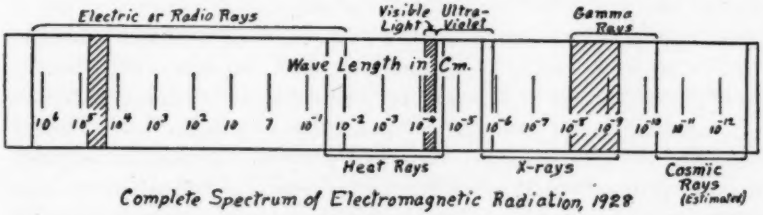
Electric or Radio Rays
Visible Light
Ultra-Violet
Gamma Rays
Wave Length in Cm.
10^{6} 10^{5} 10^{4} 10^{3} 10^{2} 10 1 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10} 10^{-11} 10^{-12}
Heat Rays
X-rays
Cosmic Rays (Estimated)
Complete Spectrum of Electromagnetic Radiation, 1928

Fig. 1. *Complete spectrum of electromagnetic radiation on a logarithmic scale. Visible light is only a small but very important part of this spectrum.*

compared. At the extreme left I have arbitrarily started the spectrum at a wave length of eighteen kilometers, which is the wave length of certain transatlantic wireless signals. There is no reason why longer waves could not be produced if desired. The electric waves continue in

* Invited paper, Charlottesville Meeting of the Optical Society.

an unbroken spectrum down to 0.1 mm, rays recently studied at Cleveland by the late Dr. Nichols and Mr. Tear. Overlapping these electric waves are the heat rays, which have been observed from about .03 cm to .00003 cm, including the whole of the visible region. The heat rays in turn are overlapped by the ultraviolet rays, produced by electric discharges; and these reach well into the region described as x-rays. The bridge between these ultraviolet and the x-rays was closed only a few years ago by Osgood's soft x-ray studies made at Chicago. Beyond these are in turn the gamma rays and the cosmic rays. Thus over a range of wave lengths of from 2×10^{-12}cm to 2×10^{6}cm there is found to be a continuous spectrum of radiations, of which visible light occupies only a very narrow band.

The great breadth of this wave length range will perhaps be better appreciated if we expand the scale until the wave of a cosmic ray has a length equal to the thickness of a post card. The longest wireless wave would on this scale extend from here to the nearest fixed star.

When the physicist speaks of light, he refers to all the radiations included in this vast range. We believe that they are all the same kind of thing, and that anything which may be said about the nature of the rays in one part of this region is equally true of the rest.

REVIEW OF EVIDENCE OF X-RAYS AS A KIND OF LIGHT

In the early studies of the properties of x-rays there appeared only one truly optical characteristic, namely that of straight line propagation. In the hands of Roentgen and his contemporaries, x-rays were found neither to be reflected nor refracted. They showed no polarization, nor could they be diffracted as light waves are. As a result, many theories were put forward as to their nature. Roentgen himself favored the idea of longitudinal ether waves. Some favored corpuscles, other electric waves of the Herzian type. Wiechert proposed, almost immediately upon their discovery, the idea that x-rays consist of rays similar to light but of much shorter wave length; yet it was many years before this suggestion found confirmation in experiment.

The first strong evidence that x-rays are really similar to light in their fundamental nature appeared when Barkla found that they could be polarized by scattering in the same way that sun-light is polarized when it is scattered by the sky. This was in 1905, ten years after the discovery of the rays.

It was eight years more before the discovery of Laue showed that x-rays could be diffracted by crystals in the same way that light is

diffracted by a ruled grating, thus establishing the wave characteristics of these rays. The striking consequences of Laue's discovery, leading as it did to undreamed of knowledge about the interior of crystals and of atoms themselves, are a matter of common knowledge.

The ordinary optical properties of refraction and reflection from polished surfaces had, however, not yet shown themselves. Though a few years after Laue's discovery evidence arose of the refraction of x-rays within a crystal, it was not until 1922 that our experiments showed the reflection of x-rays from polished glass surfaces when the rays strike the surface incident at sufficiently fine glancing angles. Only five years ago Siegbahn and his collaborators performed the first successful experiments showing the refraction of x-rays on passing through a prism of glass.

The parallelism between x-rays and light is now complete. All the distinctly optical characteristics—reflection, refraction, diffraction, and polarization, and even interference, are now recognized as properties of x-rays as well as light. Yet, because of their great difference in wave length, there remains a notable difference between these types of radiation. It will be recalled that the resolving power of an optical instrument depends upon the wave length of the light that is used. We make use of this fact in examining minute organisms under a microscope with blue light instead of white. The fact that x-rays have a wave length 10,000th that of ordinary light, makes it possible with these rays to study the detailed structure of minute bits of matter in a much more effective way than can be done with the comparatively coarse waves of light. It is partly for this reason that it becomes of especial interest to review some of the important findings of our x-ray studies and to acquaint ourselves with some of the optical problems which are now being faced in the study of x-rays.

SOME FRUITS OF THE OPTICS OF X-RAYS

"If, as Henri Poincare has said, the value of a discovery is to be measured by the fruitfulness of its consequences, the work of Laue and his collaborators should be considered as perhaps the most important of modern physics." Thus the Duke de Broglie describes the significance of the discovery that x-rays may be diffracted by crystals. Not only did this discovery show the identical character of x-rays and light, which had previously been questioned, but it also afforded a tool which has made it possible to examine the interior structure of matter with a definiteness previously almost unthinkable. By the help of this dis-

covery the Braggs analysed the atomic structure of crystals. Moseley studied the spectra of x-rays, finding from the beautiful regularity of the changes in the spectra as we pass from one element to the next, that each element has in its structure one more electron than that of the element next lighter. The measurement of x-ray wave lengths in the hands of Duane and Hunt have supplied us with what is probably our most precise method of determining Planck's constant h, which plays so important a part in the photoelectric effect, and is fundamental in the

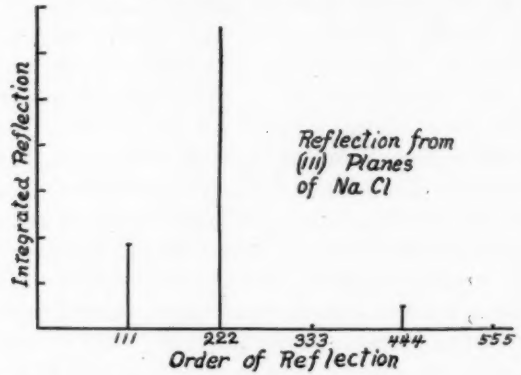

FIG. 2. *Intensities of reflection from the (111) planes of rock salt.*

quantum theory. In fact it is hard to find an aspect of atomic or molecular structure which has not been affected by the consequences of Laue's discovery of the diffraction of x-rays.

It is not, however, my desire to dwell at length on this work today, because to most of you the principle features of these investigations are already known. It may be more profitable to discuss some of the less familiar aspects of the application of the optics of x-rays.

The work of the Braggs in applying Laue's discovery to the study of the distribution of atoms in crystals is well known. The recent elaboration of their methods that has made possible a direct investigation of the arrangement of electrons in atoms is not however so familiar. The problem is essentially similar to that of the study of an optical grating. When one has a knowledge of the wave length of the light employed, a measurement of the position of different orders of the diffracted lines affords exact knowledge of the spacing between the lines of the grating. This is the optical analogue of the method of finding the distance between the layers of atoms in a crystal. It is found, however, that if the

lines of a grating are broad, the higher orders of diffraction rapidly diminish in intensity. Thus, from a study of the intensities of the orders, it is possible to estimate the breadth or diffuseness of the lines ruled on the grating. In a precisely similar manner, measurements of the relative intensity of different orders of x-rays reflected from crystals afford a means of measuring the diffuseness of the atomic layers of which the

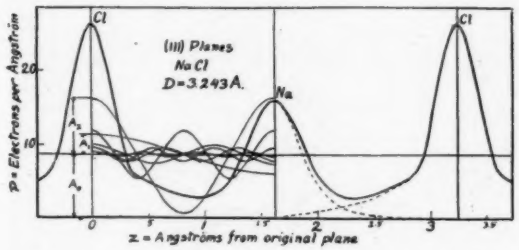

FIG. 3. *Density of electron distribution in layers parallel to the (111) planes of rock salt.*

crystal is built. This diffuseness is that due to the distribution of the electrons within the atom.

In Fig. 2 is shown, above, the relative intensity of the various orders of reflection from the (111) face of rock salt as measured by W. L. Bragg

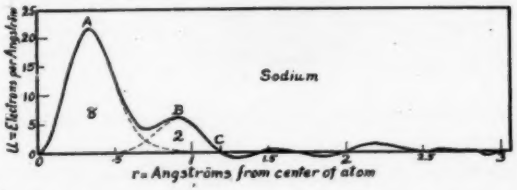

FIG. 4. *Radial distribution of electrons in the sodium ion of rock salt.*

and his collaborators. By a Fourier analysis of these intensities one finds the distribution of the electrons in the atomic layers of this crystal shown in Fig. 3. Alternate lines of sodium and chlorine atoms appear, corresponding to the alternately high and low intensities of the even and odd orders of diffraction. A more detailed examination, making use of the data from other faces of the crystal, gives the distribution of the electrons in the atoms of sodium shown in Fig. 4. It will be seen here that the radius of these atoms is of the order of 1A, which is comparable with the distance between the layers of the successive atoms. This

is one of our most direct methods of studying the distribution of electrons in atoms.

Very recently similar information has been obtained from a study of the diffraction of x-rays scattered by the molecules of a gas. You are all familiar with the time-honored prediction of rain following a ring around the moon. Such a ring is frequently due to the diffraction of the moonlight by droplets of water suspended in the air. From the size of the

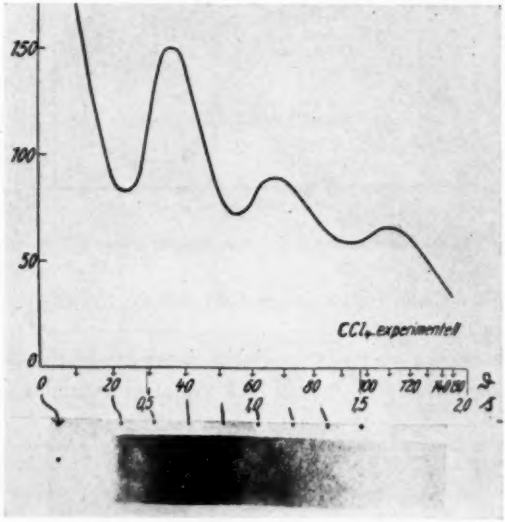

FIG. 5. *Diffraction pattern of x-rays traversing carbon tetrachloride vapor.*

ring, knowing the wave length of the light, it is possible to estimate the diameter of the water drops. In a similar way x-rays may be scattered by molecules of a gas, diffraction rings are formed, and from their size it is possible to measure the diameter of the ring. Fig. 5 shows one of these diffraction rings obtained by Debye and his colleagues, on passing x-rays through carbon tetrachloride vapor. From the size of these diffraction rings they were able to show that the distance between the atoms of chlorine in the carbon-tetrachloride molecules is 3.45A. Thus our x-ray diffraction affords a powerful means of studying the structure of molecules.

In monatomic gases the study of x-ray diffraction has also given important information. In the case of helium for example, the form of the

diffraction curves observed by Barrett and shown in Fig. 5 has revealed the form of the atom itself. When put through the mathematical mill,

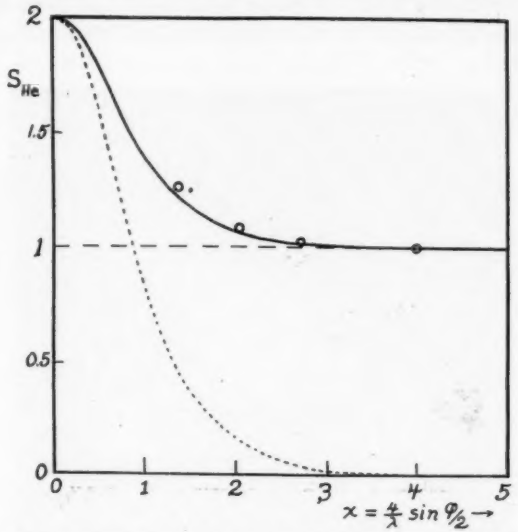

FIG. 6. *Intensity of x-rays scattered by helium for different values of x. Data from Barrett's experiments; solid line from Heisenbert atom; dotted line from Schrödinger atom.*

from these data is ground out the electron distribution in helium shown by the solid curve of Fig. 6. This result may be compared with the curve for sodium, shown in Fig. 4, obtained by a different method.

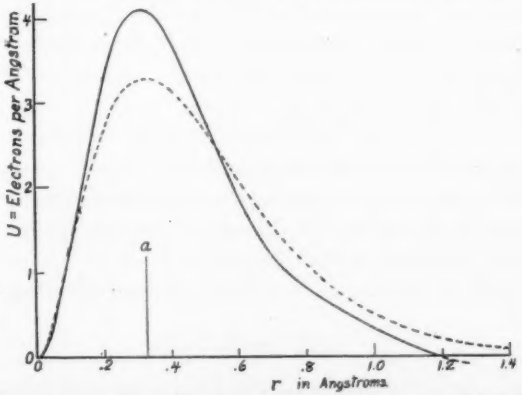

FIG. 7. *Radial distribution of electrons in atom of helium. Solid curve calculated from Barrett's experiments; broken line, Heisenberg-Schrödinger theory.*

Studies such as these come as near as is now possible to showing the inside of matter as it would be seen by an x-ray microscope. Though the nature of things seems to make impossible the invention of this mythical instrument, yet by the careful analysis of systematic experi-

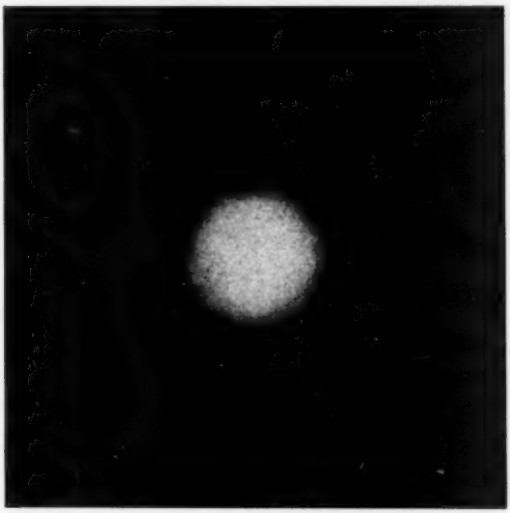

FIG. 8. *"Appearance" of a helium atom, Heisenberg-Schrödinger theory (after Langer).*

ments it has been possible to find practically the same information that such an instrument would reveal. When we thus "look" at the atom of helium we find it, as illustrated in Figs. 7 and 8, composed of a diffuse electron cloud, concentrated near the center of the atom.

CURRENT PROBLEMS OF X-RAY OPTICS

Closely associated with the analysis of the structure of crystals is the counting of the number of molecules in a cubic centimeter of the crystal. Such a measurement affords us, likewise, a knowledge of Avogadro's number and of the charge on the electron.

The method of making such a measurement is straight-forward. Bragg's law,

$$n\lambda = 2D \sin \theta, \tag{1}$$

expresses the angle θ at which an x-ray beam of wave length λ is diffracted from a crystal, in terms of the distance D between successive

layers of atoms and the order of diffraction n. The quantity n is a small integer, usually 1, while θ is an angle which can be measured with high precision. It is thus possible to determine the distance D between the layers of atoms almost as accurately as the wave length λ of the x-rays is known. The most precise as well as the most direct method of determining this wave length is by the help of a diffraction grating ruled on glass. The fact that it has recently become possible to make precise

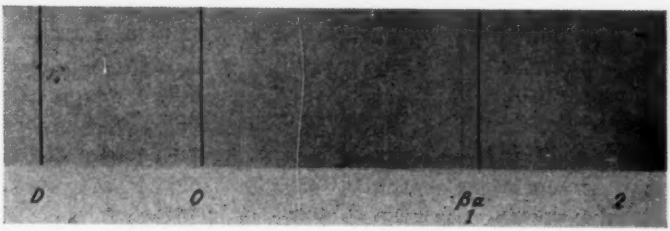

FIG. 9. *Bearden's spectrum of CuK series from glass grating of 600 lines per mm.*

wave length measurements by this method is the more interesting since the older physicists were unable to observe any diffraction effects whatever when x-rays strike an optical grating. We now find that if the x-rays are allowed to fall on the grating at a fine glancing angle, a strong reflected beam is observed which has in it the various orders of diffraction.

Careful measurements of x-ray wave lengths by this method have been made by a number of investigatiors, including Wadlund, Bäcklin, Bearden, and Cork. Typical of their results are those of Bearden, one of whose spectra is shown in Fig. 9. He finds for the $K\alpha$ line of copper a wave length of $\lambda = 1.5422$ A. This spectrum line is diffracted in the first order from a crystal of calcite at an angle of 12° 15′ 28.2″. In order to obtain a precise estimate from these data of the distance between the layers of atoms in calcite, it is necessary to apply a small correction to Equation 1 to take into account the refraction of the x-rays as they enter the surface of the crystal. When this is done, it is found from Equation (1) that the grating space of calcite is,

$$D = 3.0367 \pm .0004\text{A} \ (20°\text{C}), \qquad (2)$$

where the probable error is due chiefly to the uncertainty of the measurements of the wave length.

W. H. Bragg has shown that the distance between layers of atoms in the crystal of calcite is given by the relation,

$$D = \left\{ \frac{1}{2} \frac{M}{N\rho\varphi} \right\}^{1/3}, \tag{3}$$

where M is the molecular weight of calcium carbonate, N is Avogadro's number, i.e. the number of molecules per gram molecule, ρ is the density of the crystal, and φ is a geometric factor depending on the angle between the faces of calcite. Using the values,

$$M = 100.078,$$
$$\rho = 2.7102 \text{ g cm}^{-3},$$
$$\varphi = 1.09630,$$

which result from measurements of the usual type, and the value of D given above, we may solve Equation 3 for Avogadro's number. This gives

$$N = (6.0142 \pm .003) \times 10^{23} \text{ per mole.} \tag{4}$$

Knowing Avogadro's number, the electronic charge e is given in terms of Faraday's constant of electrolysis F by the relation,

$$e = F/N. \tag{5}$$

Using the value $F = 9648.9 \pm .7$ emu., and $c = 2.99796 \times 10^{10}$ cm./sec., we thus obtain

$$e = 4.810 \pm .002 \times 10^{-10} \text{ esu.} \tag{6}$$

It will be recognized that this result differs appreciably from the value of the charge on the electron

$$4.770 \pm .002 \times 10^{-10} \text{ esu.}$$

based on Millikan's oil drop measurements. In searching for an explanation of this difference two difficulties have come to light. First the wave length measurements made by different investigators using different ruled gratings have given results that differ by more than the apparent probable error of the individual measurements. At the present writing it would seem that this difference is due chiefly to irregularities in the rulings of the gratings, which become especially prominent in experiments of this kind, because only a comparatively small number of lines of the grating are exposed to the incident x-ray beam. It would seem that this difficulty should be removed by making a more precise

grating, and by designing the experiment in such a way that a larger portion of the grating will be exposed to the x-rays.

A second difficulty, which is more fundamental, is due to the imperfections of the crystal itself. It is known that crystals such as rock salt are much less perfect than crystals of calcite; but even calcite cannot be considered as an ideally perfect geometric arrangement of atoms in a lattice. Zwicky has recently brought forward theoretical considerations which strongly suggest that a crystal such as calcite may have extra concentrations of atoms at rather regularly spaced intervals in its lattice, sufficient to make the average density of the crystal a fraction of a percent greater than the density of the small portions of the crystal which can be considered approximately perfect. Imperfections such as those supposed by Zwicky render invalid a precise estimate of Avogadro's number according to the method as we have just outlined. Before such an application of our x-ray wave length measurements can be made with precision, it will be necessary for us to re-examine the nature of the crystal lattice and to assure ourselves that we are using a crystal that is really perfect.

This difficulty with the crystal lattice does not, however, affect the ruled grating measurements of the wave length of the x-rays. In fact it appears that by this straight-forward application of optical methods to x-ray diffraction we obtain our most reliable standard for x-ray wave lengths.

INDEX OF REFRACTION AND THE SPECIFIC CHARGE OF THE ELECTRON

Recently Birge and others have called attention to the fact that one of the fundamental constants of physics, the ratio of the charge to the mass of the electron e/m, appears to have slightly different values according to the method by which this constant is measured. Spectroscopic data such as measurements of the Rydberg constant and studies of the Zeeman Effect agree in giving a value of

$$e/m_s = 5.279 \times 10^{17} \text{ esu/g};$$

measurements of the diffraction of cathode rays by electric and magnetic fields, however, have agreed in giving a value of about

$$e/m_d = 5.303 \times 10^{17} \text{ esu/g}.$$

This difference is considerably greater than the apparent experimental error of the two types of measurement. It might be supposed that this difference represents a true difference in the properties of the electron

when it is attached to an atom. Such a proposition is however very difficult to justify on theoretical grounds, and it becomes highly desirable to find additional methods of determining this ratio which will be independent of those just considered. It appears that such a method is available, if it is possible to make a precise determination of the index of refraction of x-rays.

Acording to the Drude-Lorentz theory of optical dispersion, the index of refraction of a medium containing electric oscillators is given by the expression

$$\mu = 1 + \frac{e^2}{2\pi m} \sum_1^N \frac{n_s}{(\nu_s^2 - \nu^2)} \tag{7}$$

where n_s is the number of electrons which have a natural frequency of ν_s, N is the number of such groups of electrons and ν is the frequency of light.

If the frequency is very high, as in the case of x-rays, we may reach the condition where ν_s^2 is negligible compared with ν^2, giving, in the limiting case for very high frequency and elements of low atomic weight in which ν_s is not large,

$$\mu = 1 - \frac{ne^2}{2\pi n\nu^2} \tag{8}$$

where n is now the number of electrons per cm^3 of the refracting material. Solving this expression for the ratio e/m, and noting that

$$\nu = c/\lambda$$

we obtain,

$$\frac{e}{m} = \frac{2\pi c^2}{ne\lambda^2(1-\mu)}. \tag{9}$$

It will be noted, however, that the quantity

$$ne = \frac{Ne}{W\rho} = \frac{F}{W\rho}.$$

where N is now Avogadro's number, W the molecular weight, and ρ the density, and $F = Ne$ is Faraday's constant of electrolysis. We thus have in equation 9 a value of e/m expressed in terms of the index of refraction μ and of other quantities which are subject to precise measurements by the usual methods. We should thus be able to determine the specific charge of the electron almost as precisely as we can measure $1-\mu$.

In view of the fact that all the early attempts failed to measure any

refraction of x-rays whatever, it is not surprising that existing measurements have failed to give the precision necessary to give valuable results by this method. During the past year, however, Dr. H. E. Stauss, working at Chicago, has developed a precision method of determining x-ray refractive indices which has given results of great interest. Fig. 10 shows the arrangement of his apparatus. X-rays, after passing through

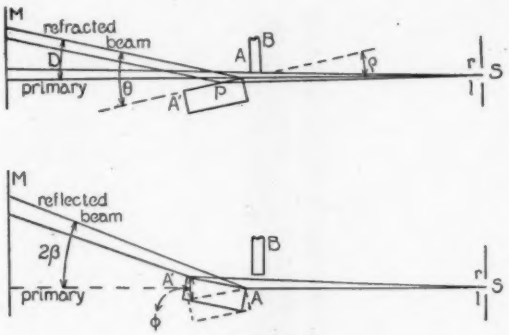

FIG. 10

a pair of slits strike one face of a right-angled prism of quartz at almost normal incidence, and are transmitted to the second face at a very fine glancing angle. Thence the rays are refracted out of the prism at an angle θ. This disposition of the apparatus is that which secures the maximum deviation of the beam, thus differing from the usual arrangement used with visible light, where the prism is placed so as to give the minimum deviation. Otherwise the problem of determining the index of refraction from the angles of incidence and refraction is precisely the same as that in the usual optical problem. In Fig. 11 is shown a dispersion spectrum of the molydenum K series which he has thus obtained.

Stauss has made measurements on the $K\alpha$ and $K\beta$ lines of molybdenum, for which he finds

$$(1 - \mu)_{K\alpha} = (1.804 \pm .001) \times 10^{-6}$$

and

$$(1 - \mu)_{K\beta} = (1.436 \pm .001) \times 10^{-6}.$$

Inserting these values into equation 9, and using Bearden's wave-length standard to calculate λ, Stauss finds from the α lines,

$$e/m = 5.290 \pm .003 \times 10^{17}$$

and from the β line

$$5.293 \pm .003 \times 10^{17}.$$

The probable error here estimated is that due merely to the uncertainty in $1-\mu$.

It will be seen that these values of the specific charge of the electron lie between those obtained from the spectroscopic and from the deflec-

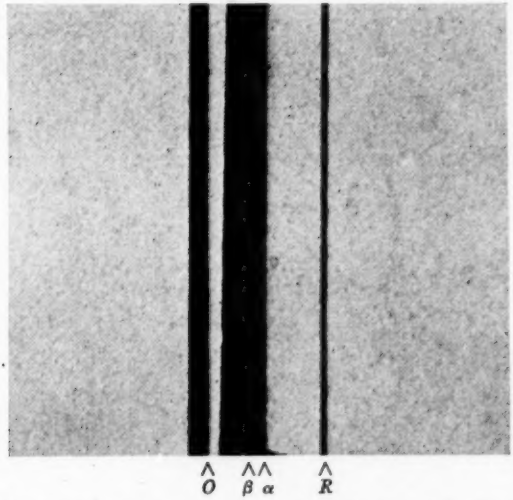

FIG. 11. *Dispersion spectrum of K series of molybdenum. O=primary beam; β and α=refracted spectrum lines; R=totally reflected beam.*

tion measurements, and it would seem that the new values are of at least comparable precision. It will be noted, however, that the value of e/m thus observed depends upon λ^2, and we have seen that λ is itself uncertain because of the probable error in the standard wave length found by different investigators. These differences in the wave length are of the order of .1%, which would mean differences in e/m of the order of .2%. A variation of this magnitude, however, would make Stauss' value of e/m fit with either of the usual values of the specific charge. It will therefore be necessary to refine our measurements of x-ray wave lengths before it is possible by this method to choose between the two alternative values of the specific electronic charge. Stauss' experiments have however indicated that we have in such an application of the methods of x-ray optics a valuable method of determining another of nature's fundamental constants.

CONCLUSION

Of the various methods of obtaining information regarding the world around us, probably the knowledge observed through the light which enters our eyes is the most important. It is surprising that so much information can be obtained through the medium of a single octave of electro-magnetic waves. It was perhaps to be expected, when x-rays were discovered, that because of their much shorter wave length they should be able to give us finer and more detailed knowledge of our world. We see how they are fulfilling this promise. We must consider x-rays as a kind of light, so that all x-ray problems are essentially problems of optics. Yet because of the differences in wave length they form a supplement to visible light which is of enormous value.

THE OPTICS OF RADIO-TRANSMISSION*

By Ernest Merritt

[Department of Physics, Cornell University, Ithaca, N. Y. Received Nov. 8, 1930.]

It will be remembered that the discovery of electric waves came as a result of the experiments undertaken by Hertz to test the electromagnetic theory of Maxwell. It is one of the cases where advance in theory definitely preceded the advance in experiment. For this reason perhaps physicists were interested in electric waves for a number of years after Hertz's discovery chiefly because of the confirmation that they gave of the electromagnetic theory of light. The older members of this audience will remember the large amount of interesting work that was done about thirty years ago in imitating the fundamental optical phenomena with electric waves several centimeters or several meters in length. Hertz himself showed that the waves were reflected and refracted like light waves. A little later their velocity of propagation was shown to be the same as that of light. They were found to show interference phenomena of various kinds. Then followed the demonstration for electric waves of diffraction by an aperture and by a grating, polarization, double refraction, rotation of the plane of polarization, total reflection, dispersion. In fact practically every important phenomenon that we think of as characteristic of light waves was found to be reproducible with electric waves. The last piece of work of this kind that I recall was published only a few years ago and used a model of the asymmetric carbon atom to give rotation of the plane of polarization of electric waves about 20 cm long.

It was while work of this sort was under way that Marconi began his efforts to utilize electric waves for radio communication—unfortunately in the beginning with only little encouragement from physicists. Naturally optical analogies were an important guide in this early work. In some of Marconi's first experiments a concave mirror was used to concentrate the rays; in a rough way his apparatus was the electric analogue of a search light. It is interesting to note that in recent years we have in the various methods of directed radio transmission an application in modern form of the same optical principles that were used by Marconi over thirty years ago.

While Marconi used optical analogies as a guide so long as they were helpful, he did not allow himself to be hampered by such analogies. He

* Presented by invitation at the Fifteenth Annual Meeting of the Optical Society of America held at the University of Virginia, October 31–November 1, 1930.

was courageous enough to keep increasing the distance between his stations until he received signals at points far beyond that at which the curvature of the earth would prevent a search light from being seen. For a result like this no optical analogue could at that time be found. And when radio transmission across the Atlantic became a reality it seemed as though there was no possibility of treating radio as a branch of optics. It is true that in a half hearted way diffraction was sometimes suggested as an explanation of how radio waves were able to bend around the earth. To some extent light waves bend around an obstacle: with the longer radio waves this bending would be much more marked. The inadequacy of this explanation was made clear by Larmor. If diffraction permits radio waves to follow the curved surface of the earth then if we reduce the wave length to that of visible light and reduce the diameter of the earth in the same proportion, we should expect visible light to bend around the surface of this miniature earth. The wave length ratio of ordinary radio waves and visible light is about 10^{10} to 1. If the diameter of the earth were reduced in this ratio we should have a sphere about a millimeter in diameter. We know that a sphere of this size casts a sharp shadow; the bending due to diffraction is too small to detect.

For a number of years after this optical methods played a very unimportant part in the development of radio. A satisfactory explanation of the bending of the waves about the earth was found by applying electromagnetic theory to a conducting sphere in much the same way that it had been used in treating the propagation of alternating currents along wires. So far as I know there is no optical phenomenon that corresponds to this. If there were we might guide light waves along wires in much the same manner that the carrier wave is guided in General Squire's "wired-wireless," and some of the problems of television would be greatly simplified!

The story of how optical methods came back into the radio field, to become more important than ever before, is both interesting and instructive. After the wartime restrictions were removed about ten years ago there was a rapid increase in the popular interest in radio. Its use as a commercial method of communication was greatly extended, broadcasting began, and an army of amateur radio enthusiasts established private stations not only for receiving but for sending. The interference between stations became so great that if radio was to survive at all as a useful method of communication some means of regulation and control became absolutely necessary. By national legisla-

tion and international agreements a distribution of wave lengths was adopted by which a certain range of wave lengths was reserved for government stations; other wave-length bands were assigned to commercial stations, to broadcasting stations, and so on. And after everything else was taken care of, what was left was assigned to the amateurs. The waves that the amateurs were left free to use were at the short wave end of the radio spectrum, for these were the waves that were believed at that time to be the least useful for communication purposes. The theory which had accounted for the bending of radio waves around the earth when optical theories failed predicted that attenuation would increase rapidly with decreasing wave length and that long range signalling could best be accomplished by longer waves than had previously been used. Up to that time experience had amply confirmed this prediction and the tendency was overwhelmingly toward the use of longer waves. The powerful stations that were built just at the close of the war used wave lengths as great as 30 km; and the belief was general that for long range communication waves shorter than one or two hundred meters were practically useless.

Since the amateurs were not allowed to use the longer waves they went ahead with undiminished enthusiasm to get what results they could with the wave lengths assigned to them. Presumably most of them were not familiar with the theoretical reasons for believing that work with short waves was not likely to prove successful; at any rate such knowledge of theory as they had did not deter them from trying experiments which the experienced radio engineer would have regarded as foredoomed to failure. When such experiments led to success with 100 m waves they tried 50 m waves and found the results still better. Gradually the wave length was reduced still further until with a wave length of about 20 m it was found possible to signal over distances greater than had ever before been reached, and this with only a fraction of the power used by the long wave stations.

So large a part of the work was done by amateur experimenters, whose results were usually not published in the scientific and engineering journals, that it is difficult to get an accurate history of this development of short wave radio. But as the result of the combined work of amateurs, radio engineers, and physicists we now know that the range of practical radio communication is a minimum for wave lengths of about 200 m and that the range may be made greater either by increasing or by diminishing the length of the waves used.

What interests us particularly, however, is the fact that the only

explanation offered for the long range reached by short radio waves is based upon ordinary optical principles. And the development of modern radio is proceeding by the application of the same methods that would be used in the case of visible light.

The bending of short radio waves around the earth is now explained as the result of refraction by ionized air. Ions are known to be present in the air even at the surface of the earth. There is every reason to believe that their number increases with the height. As the density of the atmosphere becomes less the ions move more freely and at sufficiently high altitudes it is believed that free electrons are also present. These electrons and ions influence the propagation of radio waves in the same way that they influence the propagation of light in transparent media. If we take into consideration the different conditions to which the electrons are subjected and the different order of magnitude of the waves that we deal with we are able to apply to radio all the classical theories of the propagation of light. The difficulties that arise are due to our incomplete knowledge of the conditions in the atmosphere.

Of course the greatest difficulty is that of determining how the electron density varies with the height above the earth. To compute the electronic distribution we must make assumptions as to the agencies that produce ionization, the constitution of the atmosphere at different heights, and the rates of recombination of the different ions. But in spite of the uncertainty of these assumptions there is a remarkable agreement in essentials among the different estimates of the electronic distribution. All agree, for example, in predicting that at some height above the earth there is a very great increase in the number of free electrons, and that the increase is so rapid that we pass almost discontinuously from an atmosphere that may be treated as an insulator to a region that is a relatively good conductor. This region, whose existence was suggested independently by Kennelly and Heaviside, is known as the Kennelly-Heaviside layer. A simple way of picturing the the propagation of radio waves is to think of this layer as a mirror which reflects the waves back to the earth. Since the earth also acts as a reflector in the space between two concentric spherical mirrors.

To make the picture more complete we must remember that ionized air causes absorption as well as refraction so that the medium between the two spherical mirrors is not perfectly transparent. Finally, since the reflecting layer is caused by the presence of ions, its reflecting power and also its height will depend upon the atmospheric conditions and upon the activity of the various agencies which produce ions. One of

the most important of these ionizing agencies is undoubtedly sunlight so that it is to be expected that the Kennelly-Heaviside layer will be higher, and the absorbing power of the atmosphere less, at night than during the day. Most of the well known differences in the transmission of radio signals by day and by night are explainable as a result of this difference in the height of the reflecting layer and in the absorbing power of the air at lower levels.

Although extremely useful, and in its main features undoubtedy correct, this picture is of course much simpler than the reality. The boundary of the Kennelly-Heaviside layer can scarcely be sharp enough to permit true reflection. Probably what occurs is a continuous bending of the waves by refraction throughout their whole path, with the curvature greatest at the highest point. Theory indicates also that the index of refraction of ionized air is a function of the wave length. The path followed by short waves will therefore be different from that taken by longer waves between the same two points, so that the apparent, or effective, height of the Kennelly-Heaviside layer will depend upon the wave length used.

At moderate distances from the sending station we must assume that the waves reach the observer by at least two paths. One of these paths carries the waves up to the Heaviside layer and back. The other path follows the surface of the earth. When they meet at the receiving station the two wave trains interfere, and slight changes in the effective path lengths cause the interference to be sometimes additive and sometimes destructive. This is undoubtedly one of the chief causes of the changes often observed in the intensity of radio signals, in other words "fading."

It is customary to refer to the wave which follows the earth as the ground wave and that which follows the longer route the indirect wave or sky wave. The general belief has been that the ground wave is guided by the earth in much the same way that an alternating current is guided by a wire. In the immediate neighborhood of the sending station there seems to be little doubt that this view is correct. But to what distance this mode of propagation extends is still unsettled. It seems to me that there is a good deal of evidence in favor of the view that at greater distances the ground wave also is chiefly an optical phenomenon. Certainly there are some facts which are more readily accounted for if we think of the ground wave as proceeding according to optical laws along a curved path which rises to only a relatively small height above the earth.

The chief problems in which optics can help are, however, connected with the sky wave. From the optical point of view these problems would be perfectly straightforward if we knew the conditions in the upper atmosphere. But that is precisely what we do not know. And since the region in which we are most interested is far beyond the heights that can be reached by balloons and air planes, there is at present no direct way by which we can get the needed information. Almost the only way to attack the problem is to assume that radio transmission is an optical phenomenon and then by utilizing our knowledge of optical laws to try to deduce from the observed effects what the conditions must be that produce them. This is not a procedure with which we have had occasion to become familiar. It is as though we had the problem of determining from the flickering illumination of a sheet of paper what system of lenses and mirrors between us and the source would account for the observed changes in brightness. An excellent analogy to the radio problem is that offered by the shadow bands that are seen at the time of an eclipse.

To go into the optics of radio transmission in any detail would take us entirely too far. All I shall attempt to do is to point out some of the peculiarities and some of the complexities of the problem. We have to deal not only with refraction, reflection, absorption, interference and diffraction effects, but also with magnetic double refraction and a Faraday rotation of the plane of polarization caused by the earth's magnetic field. The modern radio engineer must be familiar with the whole optical repertoir, and able to utilize his knowledge either in the design of apparatus—such for example as the radio beacon, or other directive antennas—or in interpreting observed phenomena.

The natural frequencies of the electrons in the molecule are so widely different from the frequencies used in radio work that we should hardly expect to find any of the effects which are explained in the classical wave theory by resonance of the bound electrons in the medium: such for example, as metallic reflection and anomalous dispersion. At high altitudes, however, where electrons are present in large numbers, and where their free path is long, another effect comes which gives results very similar to those of resonance. The electrons that are set in motion by the electric field of the wave will be deflected by the magnetic field of the earth and, if the direction of propagation is the same as that of the magnetic lines of force, will describe nearly closed curves. It can readily be shown that the time required for one complete rotation is independent of the speed of the electron. So that there is a resonant frequency

at which the electrons will move in spiral paths of increasing radius and will continue to absorb energy from the waves until stopped by collision. The theory of wave propagation under these conditions has been developed by Nichols and Schelleng and shows that the effects to be expected, while in the highest degree complicated, are in the main such as would be anticipated from our knowledge of the phenomena of magneto optics. The most obvious effect of the earth's magnetic field is a rotation of the plane of polarization of the radio waves. In the case of short waves this is very marked and often changes rapidly in amount as conditions change along the path of the ray. I have observed forty meter waves from a station about three hundred miles away in which the electric vector rotated continuously, requiring from ten to twenty seconds for a complete rotation, for half an hour or more. In general the waves are elliptically polarized, but the polarization often remains nearly linear for long periods. In case the waves travel across the magnetic field, theory indicates that magnetic double refraction is also to be expected. I do not know of any observations in which this double refraction has been definitely detected. But there can be little doubt that it occurs, with results that are confused by the complexity of the phenomena.

Since the magnetic field of the earth varies from point to point, the critical or resonant frequency is not the same at different points in the path. It is as though we were dealing with a broad absorption band rather than a sharp line. The crest of the band corresponds roughly to a wave length of two hundred meters. Undoubtedly that is the reason for the radical change in the behavior of radio waves when this critical frequency is passed.

In ordinary optical work the dimensions of the apparatus are usually large compared with the wave length. Even the narrowest slit is several wave lengths wide. But the apparatus used in radio work is usually small compared with the wave length. In a few cases antennas so long as to include several wave lengths are used. But this is the exception. The diameter of the coil used in the radio compass, or in any coil receiver, is often less than one percent of the wave length that it receives. This makes it possible to study the details of a radio wave pattern in a way which is hardly possible with visible light.

Recently I have been studying in this way what might be called the fine structure of a radio wave front.[1] The results are both interesting and puzzling:

[1] The results of these experiments, which were supported by a grant from the Heckscher Foundation for Research at Cornell University, will be published in detail in the near future.

The method was to use two coil receivers placed at two points on the wave front about half a wave length apart, and by means of cables to bring the signals to a cathode ray oscillograph. The arrangement was such that one signal produced a horizontal vibration of the spot while the other produced a vertical vibration, the amplitude and phase relations between these vibrations being the same as between the original magnetic fields at the two receivers. One would naturally expect that at two points on the same wave front the phase and amplitude would be the same. In this case the oscillograph figure would be a straight line inclined at an angle of 45°. Sometimes this figure was observed. But usually it was not. Ordinarily the figure was an ellipse, and an ellipse which changed from moment to moment both in size and shape. At times the changes occurred so rapidly and so erratically that it was quite impossible to follow them. Unless the conditions were exceptionally steady the results were much the same whether the signals came from a great distance or from a station only fifty miles away.

The variations of phase and amplitude were less marked when the receivers were set so that the plane of each coil was directed toward the station. In this case the received signal was chiefly that carried by the ground wave. When the coils were turned at right angles to the station direction so as to receive the sky wave only the changes in the shape and size of the figure were much more marked. On numerous occasions the signal was strong at one station at the same time that it had faded out completely at the other station less than half a wave length away.

Evidently the conditions in the wave front of a train of radio waves are not so simple as we have been accustomed to think. In fact they are so complicated and erratic that one is almost tempted to abandon the classical picture of a wave train and to attempt to build up a picture on the basis of the modern corpuscular theory. The quantity $h\nu$ is so small in the case of radio waves that even the weakest signal calls for an enormous number of photons. We might perhaps imagine an ordinary classical wave train as corresponding to an army of photons marching in regular array, each one keeping step and allignment with his neighbors. In the cases of radio reception which I have just described, where phase and amplitude vary from point to point in the same wave front, the photon army has become disorganized. Perhaps the hardships that it encountered in the upper atmosphere had weakened its morale and destroyed its discipline. The army had become a mob of photons, all marching in the same general direction, but made up of individuals and groups that acted independently.

Some picture more or less like this may ultimately take the place of

the classical picture of wave propagation. But in the case of radio waves the necessity of such a change is not yet clearly in evidence. I am inclined to think that until the change is forced upon us it would be wiser to adhere to classical methods. To introduce a radio photon into the picture before we are forced to do so might create more difficulties than it avoids. How, for example, would we get our army of photos started? Where do the recruits come from? And when they reach the receiving station what is the process by which they set up the electric currents by which we detect them? It seems to me to be one of the cases where it is "better to suffer the ills we have than flee to others that we know not of."

The observed variations of phase and amplitude along the wave front seem to me to be capable of a simple explanation along classical lines. If we assume that waves reach the receiving apparatus by several different routes it is to be expected that they will differ in amplitude, phase, and direction. An interference pattern will be formed at the receiver, which, if the conditions are steady, will consist of a fixed system of interference bands. It is quite possible that there would be additive interference at one receiver and destructive interference at the other. If we could investigate the conditions at different points in the interference pattern of a Michelson interferometer, or at the focus of a lens, we should not expect to find constancy of either amplitude or phase. If an interferometer were mounted above a hot stove the bands would probably disappear. At each instant there would still be a definite interference pattern—although probably a very complicated one. But the pattern would change so rapidly that in the case of light it could not be observed. In the case of radio waves everything is on a larger scale—even time— so that it is possible to follow the changes from instant to instant. When the conditions for radio reception are bad it seems probable, therefore, that something is going on in the upper atmosphere which is at least analogous to the convection currents in heated air. If we wish to make radio observations which involve any simple and predictable phase relations we must wait until the atmospheric conditions are favorable—just as we would have to do in the case of light.

Among the radio problems that are arousing a great deal of interest at the present time are those connected with so called "radio echoes," i.e. cases where two signals—or sometimes more than two—are received for each single signal sent. For example the dot of the Morse Code is received as two dots. At points not far from the sending station this

duplication of signals can be observed with suitable apparatus most of the time, the separation between the first signal and its "echo" being of the order of .001 sec. In such cases there can be little doubt that the first signal has followed the direct route from the station, while the second has been reflected from the Kennelly-Heaviside Layer. Observations of these echoes thus give an excellent means of determining the effective height of the layer. Sometimes more than two echoes are recorded, with such spacing as to suggest multiple reflection between the Heaviside layer and the earth. On other occasions when several echoes are observed the time intervals are such that the existence of two reflecting layers is suggested.

When signals are received from a powerful short wave station capable of transmitting over great distances another type of echo is frequently observed in which the separation between the signal and its echo is of the order of one seventh of a second. The explanation seems to be that one signal has come by the shortest route while the other has traveled all the way around the earth and reaches the receiving station from the opposite direction. The time interval between the signals has been measured with such accuracy and consistency that very plausible estimates can be made of the height at which the waves travel. Sometimes however these round the world echoes are accompanied by echoes with a retardation considerably less than one-seventh of a second—sufficiently long to permit the waves to travel two or three thousand miles, but not much further. To account for these cases the suggestion has been made that the waves reached the observer after reflection from a hillside or possibly from the waves of the ocean. Evidently the continued study of these echo signals is likely to bring most interesting results.

About two years ago still another type of radio echo was observed which presents the most puzzling problem in the whole radio field. These echoes occur with a retardation ranging from three seconds to nearly thirty seconds. Traveling with the velocity of light radio waves would cover in thirty seconds a distance of over five million miles—two hundred times the circumference of the earth! Naturally one's first thought is that the observers must in some way have been deceived, possibly interpreting some accidental noise as an echo. But the observations were made by skilled observers and with such precautions and checks that this explanation seems to be ruled out. Signals were sent out at intervals of thirty seconds from Eindhoven in Holland and were received independently by three observers, two at different stations in

Eindhoven and one in Oslo, Sweden. In numerous cases, although not in all, when an echo was observed at one of these stations it was also observed at one of the others. Frequently it was observed at all three stations, and with nearly the same retardation.

These long time echoes evidently call for very special conditions, for of the numerous attempts made to detect them only a few were successful. Two suggestions have been made to explain these echoes. One is that at times there is a stream of electrons from the sun at a distance of a million miles or so from the earth which is sufficiently dense to reflect the waves. Such an electron stream would be bent by the earths' magnetic field so as to have a tendency to reflect the waves directly back to the earth. Another suggestion is that the waves do not leave the earth at all but travel in a region of the atmosphere where the conditions are such as to make the group velocity extremely small.

It is very much to be hoped that efforts to observe these long time echoes will be continued, for if they really occur—and it is hard to doubt that they do—their study seems likely to lead to results of great importance.

SOME OPTICAL FEATURES IN TWO-WAY TELEVISION

By Herbert E. Ives

[Bell Telephone Laboratories, New York, New York, Received November 10, 1930]

A comprehensive description of the two-way television system now being demonstrated between the American Telephone and Telegraph Company building, and the Bell Telephone Laboratories, in New York City, has been published elsewhere.[1] Part of that account gives the essential features of the optical arrangements whereby the users of the apparatus are appropriately lighted, and are assured against visual discomfort from the scanning operation. Since the apparatus was first installed, however, some important changes have been made in the distinctively optical features, whereby the performance of the system has been notably improved, and its operation considerably simplified. These changes deserve description, and the present account is mainly concerned with them, although for the sake of completeness some details previously described are included.

It is an inherent feature of the two-way television system that either user is continuously scanned as he views the image from the distant station. The beam scanning method,[2] by which a beam of light sweeps over the subject's face, enables the scanning operation to be performed with a minimum amount of light. Even so, because of the relatively low intensity of the television image, it is necessary to reduce the intensity of the scanning beam in every way possible. In the two-way apparatus as first operated, advantage was taken of the fact that the photoelectric cells employed, which were of potassium, were principally sensitive to blue light. The scanning beam derived from a high power arc lamp was accordingly passed through a deep blue filter, which reduced the photoelectric efficiency of the beam very little, but because of the relatively low visual value of blue light, effectively reduced the brightness of the beam many times. The user of the apparatus saw, above the incoming image, merely a mild blue spot of light, which did not interfere with his vision.

A disadvantage of the use of blue light, which was anticipated, and found in practice to be quite real, was that dark, tanned, or ruddy complexions were rendered as altogether too dark, in comparison with whites such as the ordinary linen collar. The effect is precisely that encountered in the earlier photographic processes before color sensitive plates

[1] Bell System Technical Journal, July 1930, 448.

[2] J. O. S. A. & R. S. I., *16*, 177; 1928.

and color filters were available. While this defect was minimized by the use of a dark background, and to some extent by chopping off the highlights by electrical means, it was recognized as undesirable.

One recent improvement in the apparatus is a change in the nature of the scanning light, whereby, without sacrificing the general principle

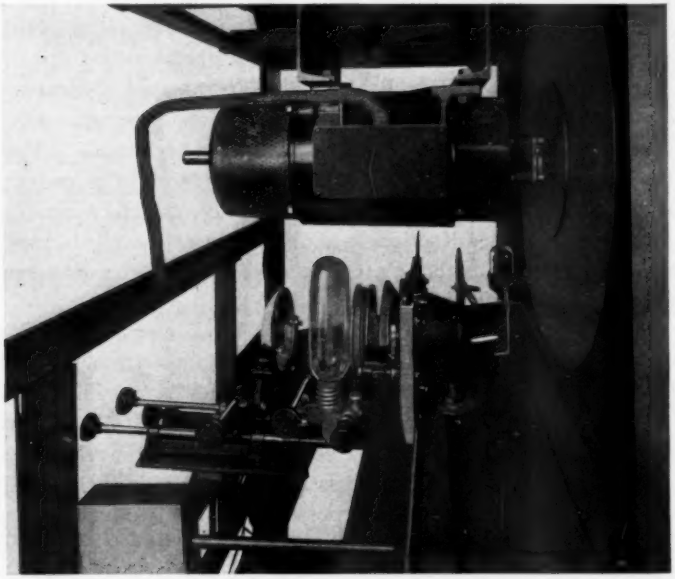

FIG. 1. *Incandescent lamp used for scanning light.*

of using visually inefficient but photoelectrically efficient radiation, the proper balance of tone values in the face is restored. This has been accomplished by adding to the battery of blue sensitive potassium cells, a group of red sensitive caesium oxide cells, and scanning by *purple* instead of blue light, that is, both ends of the visible spectrum are used in place of one end.

In making this change, a number of others were involved, most of which resulted in simplification or improvement. One important alteration was the substitution for the arc lamps previously employed, of incandescent lamps of a type available from motion picture projection practice, as shown in Fig. 1. The lamp employed has for its radiator, four vertical helical coils of tungsten wire, and is furnished with a re-

flector which images the coils back on the intervening spaces. An efficient condenser system throws a brilliant rectangular image on the back of the scanning disc, which is substantially uniform over the whole field. With this unit, the scanning beam as it leaves the projection lens is somewhat larger in diameter than the beam as produced from the arc. Con-

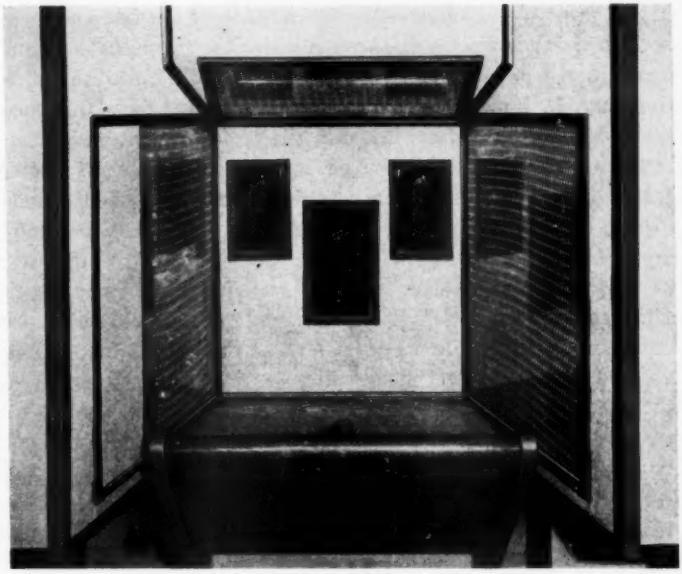

FIG. 2. *Interior of two-way television booth showing location of two caesium cells above and to either side of scanning and viewing aperture.*

sequently, for positions away from the focused image of the disc holes, the scanning beam is larger than before, with some resultant loss in the range of sharpest definition. Since, however, the user of the two-way apparatus is seated in a fixed chair, he has little opportunity to move far out of the plane in which the disc holes are focused, so that this objection is not serious. The advantages of this substitution are two-fold. First is a great gain in simplicity of operation and maintenance. Second, the incandescent lamp, being a lower temperature radiator, radiates relatively many times as much red light as does the arc, for the same amount of blue. Consequently, once an incandescent lamp unit was found which gave the amount of blue light required for the potassium cells, the great excess of red light made possible the use of

relatively few caesium oxide cells. Since these are intrinsically somewhat more sensitive than the potassium cells, the net result was that a red signal comparable with the blue signal could be added by the installation of only two caesium cells, each of less than half the electrode area of the potassium cells.

It was found most convenient to mount the two caesium oxide cells directly in front of the observer, to either side of the microphone, and above the opening in the booth through which the scanning beam enters, and through which the incoming image is seen. This arrangement is shown in Fig. 2. The only objection to placing the cells in this position is that they encroach somewhat into the region where images of the cells (which are virtual light sources) are likely to be seen reflected in eyeglasses. Since, however, the head is normally directed somewhat downward, cells placed in these upper corner spaces are not serious offenders in this respect.

Two other features of the two-way system which needed revision when the caesium cells were adopted, were the variable angle prisms used to direct the scanning beam upward or downward, depending on the user's height; and the general illumination of the television-telephone booth. As to the variable angle prisms, the only change called for was the substitution of achromatic prisms, corrected for deep red and blue light, in order to prevent the scanning beam from breaking effectively into two beams for large angles of deviation. The problem of general illumination of the booth is principally the choice of a color of light which shall affect neither the potassium nor the caesium cells. For this purpose, a monochromatic yellow-green was chosen, secured by covering all the lights with a combination of orange and signal green glasses. The potassium cells are insensitive to this color of light, and the caesium cells were rendered so by placing over them, windows covered with a deep purple gelatin. This choice of illumination color made possible a satisfactory general level of illumination of the booth and the surroundings of the image without introducing spurious signals.

The transmissions of the purple filters, the response curves of the potassium and caesium oxide cells, the radiation curve of the incandescent lamps used for the scanning beam, and the transmission curves of the glasses used over the lamps for general illumination, are shown in Fig. 3. Comparing these with the response curve of the eye, also shown in the same figure, it will be evident how the general problem of securing photoelectric signals of maximum efficiency without interfering with the general quality of the image, or desirable conditions of illumination, has been secured.

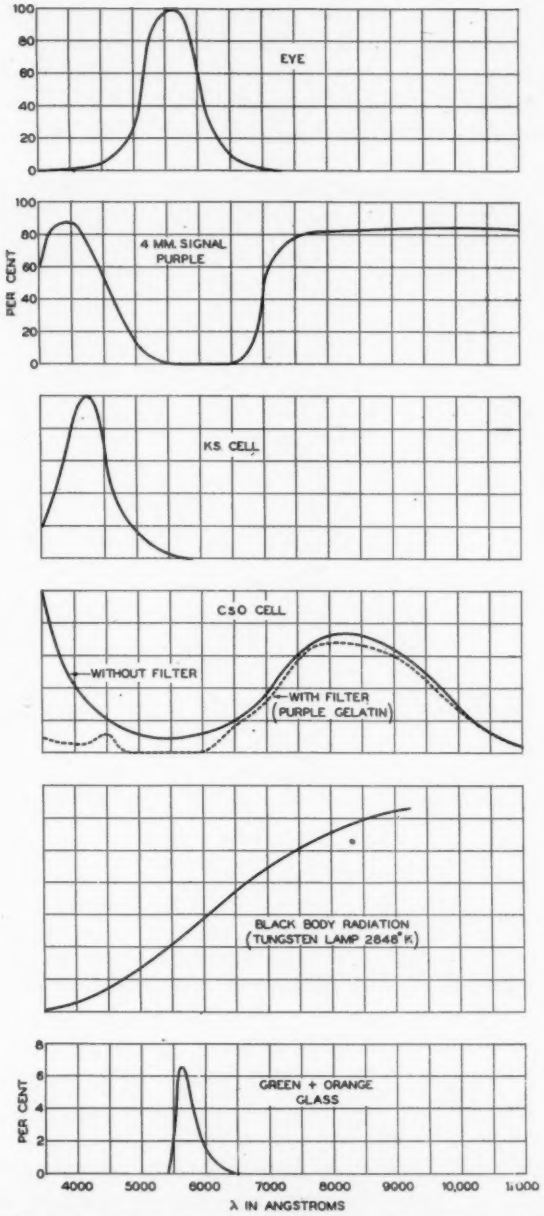

FIG. 3. *Spectral characteristics of the scanning, viewing and illumination elements in two-way television system.*

Before going on to describe some of the optical features at the receiving end, we may pause to discuss the improvements in the television signal which have been introduced by the changes just described. There is, of course, a substantial gain in the steadiness of the image due to the elimination of the arc lamps, much of whose effective radiation was from the arc stream which always wanders somewhat. The chief gain, however, is in the tone quality of the image of the face. The difference is very clearly shown if shutters are arranged so that either the potassium or the caesium cells may be used alone, alternately, and can

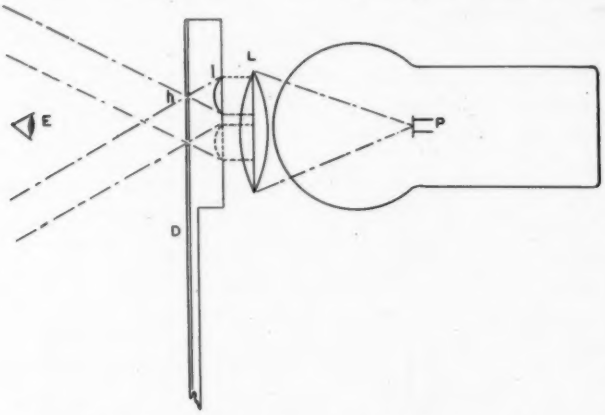

FIG. 4. *Section of disc with lens system for utilizing small area light source.*

then be quickly exposed together. With the potassium cells alone, as already noted, flesh tints are in general too dark, and tanned or ruddy complexions show unnatural contrast with the whites. Highlights due to reflection on the skin are often observed to be out of scale, with a resultant effect of mottling of the skin. With the caesium cells alone, on the other hand, the flesh tints are in general too light, and faces are apt to appear very flat. These differences were anticipated, but others not so obviously to be expected, have been observed. For instance, with the caesium cells, the pupil and iris of the eye are brought out with rather startling blackness, while with the potassium cells, the detail around the eyes is apt to be lost. The most satisfactory results are obtained with both sets of cells acting, for, as was hoped, the combination of the two ends of the spectrum, gives, in the case of the face, an effect very like that which light from the middle of the visible spectrum would give, that is, an "orthochromatic" image, as it would be described in

photography, while the definition of important points, such as the eyes, is distinctly improved.

Passing now to the receiving end of the two-way television apparatus we recall that in the apparatus as originally set up and described, a

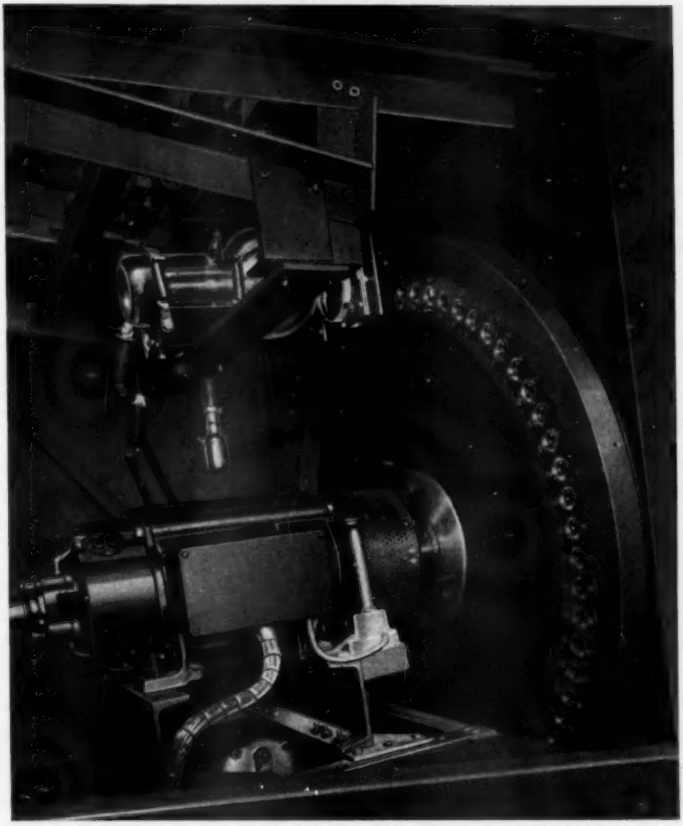

FIG. 5. *Disc with condensing lenses as used at the receiving ends of two-way television system.*

simple disc with a spiral of holes was used, immediately behind which was a neon lamp with a large flat water-cooled electrode. On continued operation, it was found that the heavy current demanded in these lamps, in order to secure an image of sufficient brightness, caused rapid sputtering on the closely adjacent glass wall, necessitating frequent re-

newals of lamps. A very radical change in the disc and lamp design has been made by which this undesirable situation has been remedied.

The change in the disc consists in substituting for the simple Nipkow disc, with its spiral of holes, an alternative form, suggested also by Nipkow, in which each disc hole has associated with it a condensing lens, positioned so as to focus, in combination with a fixed collimating lens, an image of the source on the disc hole. The optical arrangement is shown in Fig. 4, and a photograph of the disc with lenses and lamp in place in Fig. 5. Referring to Fig. 4, D represents in section the simple disc with a spiral of holes, h; l represents a small short focus lens, fixed in position with respect to h at a distance equal to its focal length; L represents a fixed lens of diameter large enough to cover the entire frame of the picture and the lenses l; P represents the glow lamp electrode. A great advantage of this optical arrangement is that the cathode of the glow lamp can be made quite small, and can be removed, as shown, to a considerable distance from the glass wall of the containing tube. In consequence of these changes in lamp design, a very high current density can be obtained for a relatively low expenditure of energy, with at the same time a long lamp life.

The condenser lens disc is observed exactly as the simple disc, by the eye placed at E. According to Nipkow, when lenses are used on the disc, the holes should be covered with diffusing material. This is not necessary in the present case, because in the two-way booth, the observer has very little latitude of motion, and it is only necessary that his eyes lie in the overlapping cones of rays from the extreme holes in the field. By making the lenses l of large diameter compared with their focal length, the solid angle through which an image is visible is entirely adequate.

The general characteristics of the lamps used are shown in Figs. 4 and 5. The cathode is a heavy slug of copper, into which a hollow cylindrical aluminium electrode is screwed, shielded from the copper by mica and glass. Because of the large mass of the copper, the water-cooling is no longer necessary. With lamps of this type, the amplifier circuit used before makes it possible to obtain images of much greater brilliancy, whereby the contrast between the image and the scanning light is still further increased beyond what was before found satisfactory. This margin of brightness is so large that it has been found possible to use lamps filled with helium in place of neon, giving a much whiter image, more pleasing to some people.

REFLECTING SCREENS FOR RELIEF PICTURE PROJECTION

By Herbert E. Ives

[Bell Telephone Laboratories, New York, New York.
Received November 7, 1930]

If a series of pictures, taken from juxtaposed points of view, are projected from a battery of projectors onto a reflecting screen, the projected picture will exhibit relief, provided the screen has certain properties. These properties are (1) that of reflecting the light beams directly back toward the projectors, with no lateral spread, and (2) that of introducing a vertical spread or diffusion. Two types of reflecting screens having these properties are described. One consists of vertical rods, of front and rear curvatures such that the image of the projecting lens formed by the front surface is accurately in focus on the rear surface, which is coated with diffusing material. The second type consists of vertical strips of mirror, standing in pairs at 90° to each other and alternately plane and horizontally ribbed.

In an earlier paper,[1] I have discussed a method of projecting pictures in relief which depends for its success upon two characteristic features:

Fig. 1. *Projection in relief using a battery of projectors P, and a unidirectional reflecting screen S.*

First, simultaneous projection of a large number of appropriate views from a battery of juxtaposed projectors; second, the use of a translucent screen placed between a pair of opaque line gratings, whose function is to isolate a separate picture for each direction of viewing.

I propose to describe now two forms of projection screen, intended to

[1] J. O. S. A. and R. S. I. *18*, 118; 1929.

be used in the same way as before,—with a battery of projectors—but for reflected instead of transmitted light. Aside from this difference, the new screens have the advantage of a very complete utilization of the incident light, as contrasted with the large loss incurred with opaque line gratings.

By reference to Fig. 1, the essential nature of a reflection screen for this purpose, can be readily grasped. At P are the juxtaposed projectors, each throwing upon the screen S a separate picture. Each picture has been made in a camera whose position with respect to the original object was approximately the same as that of projector and screen. Now a simple flat screen will exhibit, under these conditions, only a confused jumble of images. What we must have is a screen which will reflect only one complete image for any chosen direction of observation. Further, if the images are to correspond to the original object for all directions of observation each image seen must be the one projected along that same direction. In short, the screen must have *the property of reflecting the light back upon its source.* This statement is true however only with respect to the horizontal plane, in which the two eyes lie, and in which the observer can move. Vertically the screen must diffuse light like an ordinary reflection screen, otherwise the observer (if he could stand in the path of the light without obstructing it) would merely see a reflection of the projection lens. At first thought, one might imagine that the kind of screen needed is, aside from the property of vertical diffusion, practically a concave mirror; for the action of the postulated screen is in a sense a focussing action. It is in fact true that a concave mirror (or a lens if transmitted light is used) on whose surface our projectors focus their "slides," would form a conjugate image of the battery of projectors, and that the eyes placed in this conjugate image would see a relief picture at the surface of the mirror. If, however, an eye is moved out of the conjugate position, say to a point nearer the mirror, the composite image now seen corresponds to a position of the eye farther away from the original object, instead of closer to it. In other words the conjugate positions of object and image are only properly disposed at the focus of the image formed by the lens or mirror, and vary in an inverse direction away from the focal point. Our postulated reflecting screen must, unlike the mirror just discussed, have the property that when the eye moves closer to it the image seen corresponds to a position of observation closer to the original object.

The first method of making such a directional reflection screen is to

impress upon a sheet of transparent material a series of vertical cylindrical lenticular ridges, of such curvature as to focus the (substantially) parallel beams of light from the projectors upon the rear face of the sheet. The rear face in turn is formed into curved ridges accurately opposed to those of the front face, of such curvature that the rays from the front ridge are sharply focused for all directions of incidence. The rear surface is then given a coating of white diffusing paint.

The mode of action of this double ridged structure is shown in Fig. 2. Here b_1 and b_2 are two bundles of parallel rays each coming from a separate projector, striking one of the vertical ridges. They each focus as a narrow vertical line on the diffusing reflecting layer m on the rear ridge. Let now an observer station himself in front of the screen. It is clear, remembering that these ridges are quite small—that is of width comparable with the diameter of the pupil of the eye—that he will only see the illuminated rear surface if his eye is placed in the path of the beams b_1 or b_2. In the horizontal plane no other light can reach his eye from the rear ridge. In the vertical plane, due to the diffusion of the light up and down he will see the whole height of the screen illuminated. Imagine a single projector illuminating all the ridges together making up the screen. If an observer places himself exactly above or below the projection lantern, he will see the picture in the projector displayed upon the screen. If he moves to either side the picture will disappear. If, however, the whole battery of projectors is acting, the observer will see a different picture for each (horizontal) position of observation. The projected picture will accordingly be different for each eye, and if the proper pictures are placed in each projector, will exhibit stereoscopic relief.

That this form of screen will function properly if the eyes are placed vertically above or below the projectors is sufficiently obvious. It remains to make clear that, unlike the concave mirror discussed above, it also functions for any position of the observer. This is illustrated in Fig. 3 where the eye e is imagined placed at a point closer to, and to one side of the normal to the screen, S. The double headed arrows here represent rays reflected to the eye, the single headed arrows the corresponding projected beams. It is evident that as the eye moves closer the rays received by it correspond to those which would have entered an eye closer to the original object (occupying the same position relative to the original battery of cameras as the screen does to the projectors). This is to be contrasted with the paths of rays from a mirror M, which are indicated by the dashed lines, originating at a conjugate point e'.

One further point: It is clear that the image seen by an eye at any point except above or below the projectors is a composite one; each reflecting element in general sends to the eye a strip from a different projector.

The exact form of the surfaces necessary for this unidirectional reflection may be derived from Fig. 2. If the front cylindrical ridges are

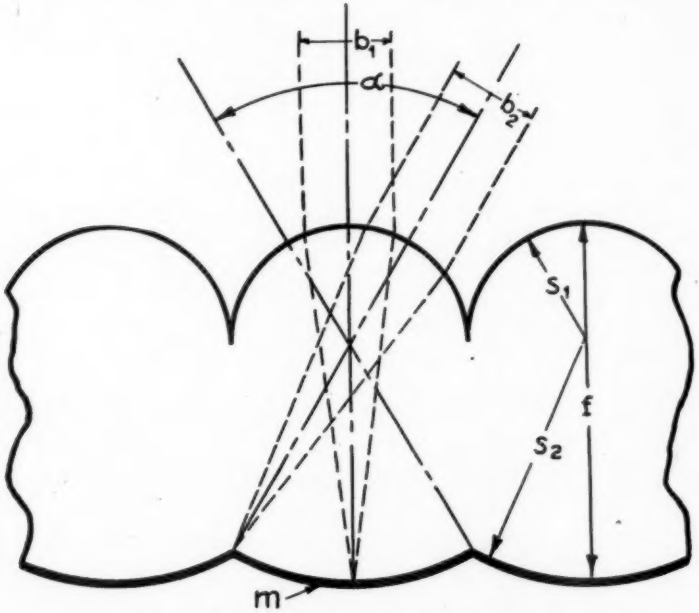

FIG. 2. *Cross-section of unidirectional reflecting screen of the "rod" type.*

made with a radius of curvature S_1, and the medium has a refractive index n_1, simple lens theory gives for the thickness, assuming the rays outside the medium to be parallel,

$$f = S_1\left(\frac{n}{n-1}\right).$$

The radius of curvature of the rear cylindrical ridges will be

$$S_2 = f - S_1 = \frac{S_1}{n-1}$$

the front and rear ridges having the same axis. For a refractive index of 1.5 the thickness f, will be three times, and the rear cylinder radius

twice, the front cylinder radius. For a refractive index of 2, the radii will be the same; i.e. the structure will consist of portions of cylinders of circular cross-section. The angle d, between the axes of the extreme bundles of rays focused on the rear cylinder determines the angle through which the relief picture can be produced. For the case of refractive index 1.5 the angle α is 60°. The angle increases with refractive index, and would be 180° for refractive index 2, except for the practical limitation imposed by the screening action of the ridges on each other.

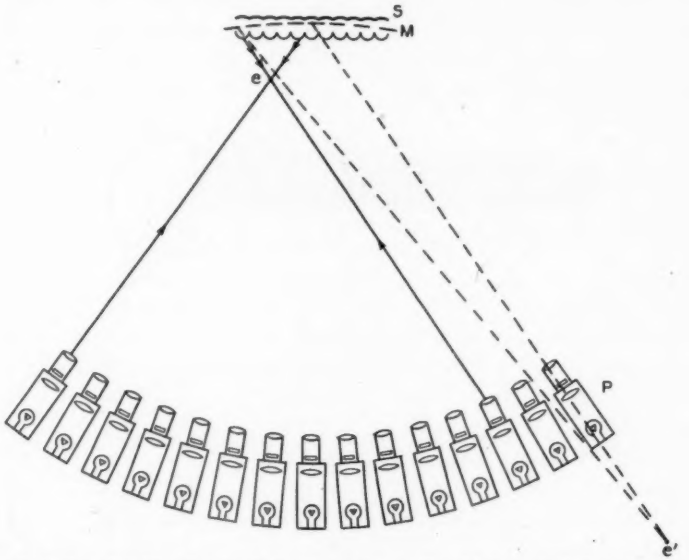

FIG. 3. *Action of unidirectional reflecting screen for eye placed between projectors and screen*

Before describing the second form of directional reflecting screen some account may be given of practical means for achieving this first form. As just noticed, if a medium of refractive index 2 is available the front and rear ridges are of the same axis and radius. This suggests the construction of a screen of rods of circular cross-section. The material most suitable would be glass, which can be drawn of very accurate circular form. A feasible construction, if rods can be obtained of refractive index 2, is to place a large number of these side by side in a flat frame, painting their rear surfaces with white or metallic (e.g., aluminum) paint. For ordinary viewing distances, rods 2 or 3 millimeters in diameter would be suitable.

If material of this high refractive index is not available, rods or their equivalent, of the cross-section shown in Fig. 2 must be formed. While glass rods could be ground to this shape, it is more feasible to use some material which can be machined, or extruded, such as celluloid. Fig. 4 shows in perspective a rod fabricated from 1/8″ celluloid sheet, of refractive index 1.5. Strips were first cut from large sheets, each 2′ long and slightly over 3/16″ wide. The two original faces were then sandblasted to give, after painting black, a non-reflecting wall. One edge of the strip was then given a cylindrical face of 1/16″ radius in a

FIG. 4. *Perspective view of rod element of unidirectional reflecting screen.*

milling machine using a high speed slow feed cutting tool, the other a cylindrical face of 1/8″ radius, the total thickness being made 3/16″. The front (small radius) surface was then carefully polished, the back surface coated with aluminum paint. The complete screen was composed of 200 of these, making a screen 2′ by 2′.

There are certain limitations to screens of this type, which, while not serious, deserve consideration. The first is that the screen must be so tilted about its horizontal axis that first surface reflections of the projection lantern lenses are avoided for all positions of the spectators. The second is that there is inevitably a certain amount of scattered light, which cuts down the contrast of the picture, due to multiple reflections in the rods, which superpose faint images from the other projectors on the main image, which alone has been considered in the analysis of Fig. 2. The third limitation is that when a rod is viewed at angles far from the normal to its axis, the distance from front to back is increased, and the curvatures of its faces are no longer correct. Thus if the observer's eyes are on a level with the bottom of a large screen which he views normally, the top of the screen, which he views at an angle, will no longer give the sharp directional reflection demanded. This defect can in part be met for a limited range of observing positions by making the rods progressively thinner toward one end. A fourth limitation is that the image formed by the cylindrical front surface of the rod exhibits "spherical" aberration, so that rays far from the axis are not accurately focused in the rear faces. This error becomes greater

as the aperture of the rod is increased. It may be corrected by covering the front faces of the rods from either side, but at the cost of a diminished angle of visibility of the picture.

The realization of these limitations to the "rod" form of screen has led to the study of another form, in which the paths of the light rays are

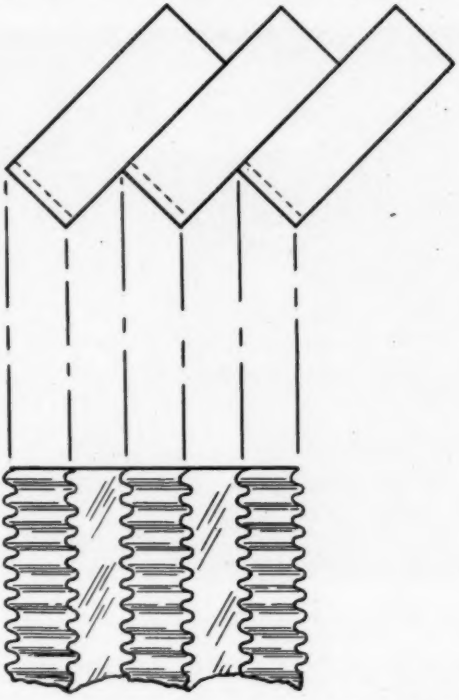

FIG. 5. *Form of unidirectional reflecting screen involving no refracting elements.*

controlled entirely by reflection, without the interposition of refracting media. Each element in this screen consists of two vertical reflecting surfaces, at right angles to each other, one of which is provided with a unilateral (vertical) diffusing structure. The two mirrors at right angles to each other constitute a well-known optical arrangement for returning a ray upon itself. The unilateral diffusing structure is merely a series of horizontal ribs which reflect and diffract the incident light upward and downward, without altering the direction of regular reflection in the horizontal plane. The exact form of the reflecting screen is shown in Fig. 5. It is built up of vertical strips of ribbed glass, whose sides

are ground accurately at right angles to the ribbed face. A series of such strips, silvered and polished on the ribbed and plain sides are "stacked" against each other to form strips, standing at 45° to the plane of the screen as a whole. Light striking either face of the V's thus formed is reflected to the other face, and so back toward the projector, meanwhile

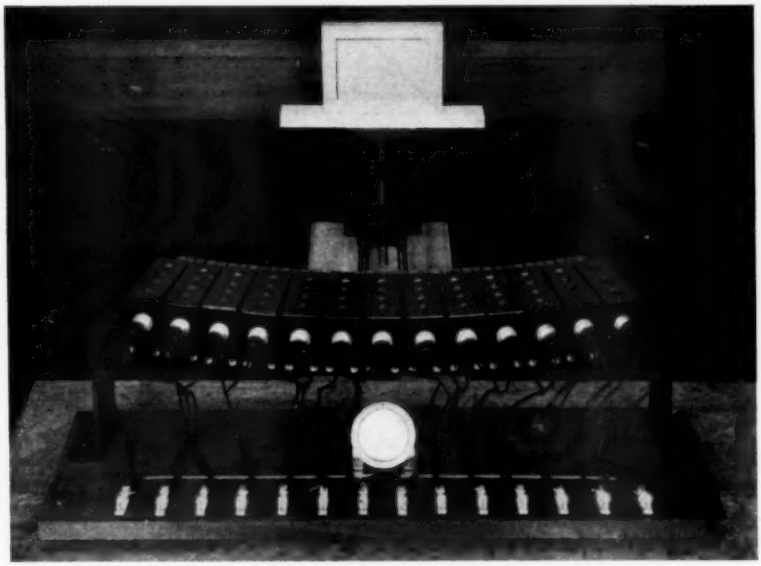

FIG. 6. *Battery of projectors and unidirectional reflecting screen as used in relief projection.*

experiencing at one reflecting face or the other diffusion in the vertical direction.

This form of screen depends, for its satisfactory functioning, on the 90° angle between the two faces of the V being highly accurate, which, for the construction shown, depends on the accuracy of the cutting and grinding of the constituent strips, and on their lying accurately against each other. While a delicate task to build this kind of structure, it is no more impractical than many which have found wide use in industry, for instance, the many bladed rotors of turbines.

It is of some interest to point out that this kind of screen, like the rod screen, has certain limitations as the angle of incidence of the rays is varied. Consider for the moment a large sheet of ribbed glass, silvered on the front. If this is set up with its horizontal axis at 45° to the eye

and to a point light source, the reflected image of the light will be seen stretched out into a vertical line,—which is one of the phenomena we depend on in the present case. If the eye and the light source are equally distant from the mirror, this vertical line is straight. If, however, these distances are not the same, this line becomes curved, with its convexity in one direction if the eye is closer to the mirror than the light, in the opposite direction if the light is nearer. Now this condition will occur in the projection screen for observing positions far removed from the

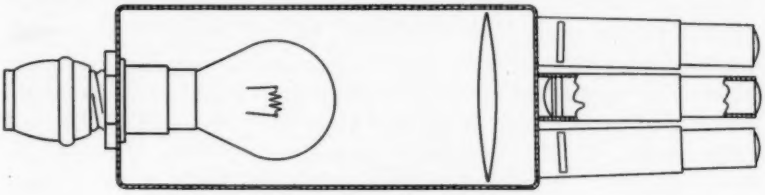

FIG. 7. *Detail of projector unit.*

projectors. It results in a distortion of relief in parts of the picture, for instead of the eye perceiving its proper image, it sees part of the image from another projector corresponding to a somewhat different point of view. This kind of distortion is quite different from the blurring that the rod screen introduces at large angles of observation, and less noticeable. Since the reflection screen is free from the haze of multiple reflections and need not be tilted to get rid of confusing reflections, it appears on the whole the preferable form, although more difficult to construct.

Of the several designs of directional reflection screen here described, an experimental test has been made of two of the "rod" type. The first was in the form of a celluloid sheet, 5″ by 7″ in size, impressed with opposed cylindrical ridges, 1/50″ wide, in an embossing press. The second was the built-up screen, 2′ by 2′, described in detail above. In order to test these screens, a multiple projector was built, a photograph of which is shown in Fig. 6. It consists of 13 units, each of which has three adjusted juxtaposed projection elements, the whole set of projectors being mounted on a single base plate. Details of each triple unit are shown in Fig. 7. The "slides" for projection are contact prints on pieces of lantern slide plate, printed from 39 negatives made in succession with a camera pivoted about the object. Each slide is inserted and separately adjusted in register with its neighbor, by projection on a flat screen. (Printing all the pictures on a single long film is

an indicated improvement.) The ridged screen is then substituted, and the confusion of images resolves itself into a relief image.

The general method of relief picture projection, which has been discussed in this paper, constitutes a definitely successful solution of the problem of throwing on a screen pictures which show relief without the use of any special spectacles or other auxiliary apparatus at the eyes. The great practical objection to the method is the large array of projectors required if a group of spectators of any size is to be accommodated. Allowing 2″, or somewhat less than the interocular distance, from center to center of the projectors, an auditorium 50′ wide would need 300 projectors in order that everyone in the cone limited by the sides of the screen and the extreme projectors would be able to see the picture. If the screen is to display a moving picture, this calls for a corresponding number of motion projectors and films. In order for the method to approximate to commercial practicability some means for placing the images on a single film and diverting the projected rays in turn to each direction of projection would appear worthy of study. This, of course, if the optical and mechanical means could be worked out, would call for very high speeds of projection, and intense light sources, probably quite beyond anything now in sight. On a very modest scale, something might be done by feeding the battery of projectors a single film passing through from left to right continuously, and having its pictures in groups of the number of projectors. As each group came into position, so that each projector had its appropriate picture, an instantaneous exposure could be given by, say, a high power spark. But the development of this phase of the problem may well wait on physical and technical advances not yet clearly visioned. For the present, it is sufficient to emphasize that the first stage in the problem, namely, the actual accomplishment of relief projection, has been successfully demonstrated.

CONTRIBUTION TO STUDY OF EFFECT OF ELLIPTICAL POLARIZATION UPON ENERGY TRANSMISSION

By R. V. Baud

[Research Laboratories, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.
Received October 10, 1930]

ABSTRACT

This paper contains the equation for the energy transmission in a polarized light equipment that is composed of lenses, two crossed polarizing prisms, two crossed mica plates in 45° position with respect to the prisms, and a light source that produces "white" light. The present study of the energy transmission is primarily a continuation of previous photoelastic researches, see references. However, it is believed that the equation given will be of interest and practical use not only for workers in the field of optical stress analysis, but also for investigators working with the same equipment along other lines.

Statement of the Problem

In a previous issue of the Journal[1] a new method of carrying out photoelastic experiments was outlined, and it was suggested that use be made of it in cases where simplicity is considered as important as accuracy. It consists of recording the isochromatic lines of a model subjected to external forces, and a strip made of the same material as the model, subjected to pure bending. By matching the shades of model and strip, a complete record of the distribution of stress is thus obtained in a very simple manner.

The equipment required for such experiments consists of lenses, two crossed polarizing prisms, two crossed mica plates in 45° position with respect to the prisms, and a light source. The source that is most readily available is one that produces "white" light. The first polarizing prism makes this light plane polarized. The first mica plate of which one of the axes is denoted by *MPP* in Fig. 1, transforms the plane polarized light rays into elliptically polarized rays, and not in circularly polarized ones, as is sometimes assumed erroneously.

Circular polarization is a special case of elliptical polarization and is obtained for rays that have a wave length λ of such magnitude as to satisfy equation (1):

$$\frac{2n+1}{4}\lambda = (n_1 - n_2)d \qquad (1)$$

in which n is an integer, d is the thickness of the mica plate, and n_1 and n_2 are the indices of refraction for the particular wave length λ in

[1] *Further Development in Photoelasticity*, R. V. Baud, J.O.S.A. and R.S.I., *18*, 5; 1929.

question. For these rays the mica plate is a true quarter-wave plate, but not necessarily a "one quarter" wave plate as is seen from equation (1). The wave length λ for which the plate produces a retardation of one quarter the wave length, i.e. $n=1$, is further on denoted by λ_1. In practice the plates are commonly so selected with regard to thickness that $n=1$ and approximately $\lambda_1=5890$A (Sodium).

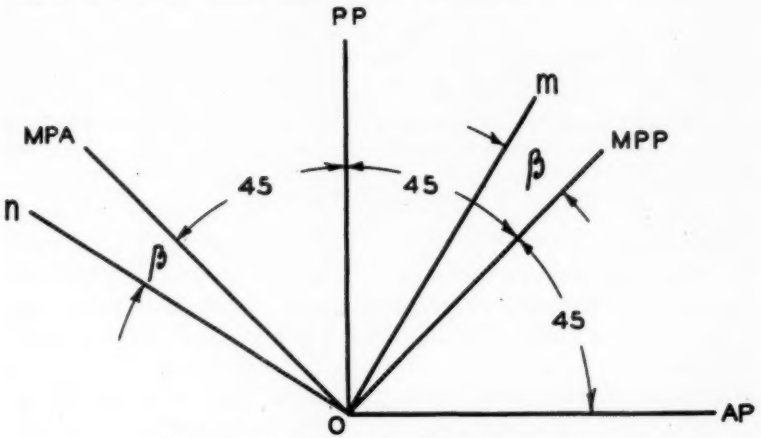

FIG. 1. *Principal planes in polarized light equipment.*

PP—polarizing prism. *AP—analyzing prism.*
MPP—mica plate polarizer. *MPA—mica plate analyzer.*
m, n—axes of refraction.

It is to be expected that due to these conditions the color pattern will not only depend upon the difference between the indices of refraction of the material to be tested but to some extent also upon the positions of the axes of refraction relative to the position of the axes of the mica plates, i.e., upon angle β in Fig. 1, as well as on the deviation from a true quarter wave retardation. The problem before us consists therefore of obtaining a quantitative expression for the phenomenon described, a problem that lends itself to both an analytical and experimental treatment. This paper contains part of the results so far obtained, namely the energy equation for the instrument with the optical parts set as described above.

EQUATION FOR THE ENERGY TRANSMISSION

If the mica plates were true quarter wave plates for each and every one of the various wave lengths λ', the final energy E transmitted

through the instrument would be as derived in a previous publication,[2] and as given by equation (2)

$$E = E_0 \sin^2 \frac{\Phi'}{2} \tag{2}$$

in which E_0 is the corresponding energy incident upon a small area of the specimen and ϕ' is the phase shift produced by the specimen for the wave length λ', measured in radians. If this shift is given in the same units as the wave length λ' as is customary, and denoted by R', equation (3) has to be used to transform it into radians:

$$\Phi' = \frac{R'}{\lambda'} 2\pi. \tag{3}$$

As mentioned in the previous publication,[3] a factor f has to be inserted in equation (2), because the mica plates are not true quarter wave plates for the various wave lengths λ'. Equation (2) then reads as follows:

$$E = fE_0 \sin^2 \frac{\Phi'}{2} \cdot \tag{2a}$$

The quantitative expression for f that has been derived by the writer is given by equation (4)

$$f = \cos^2 2\beta + \sin^2 2\beta \cos^2 \Delta \tag{4}$$

in which β is the angle between the axes of refraction m, n and the axes of the mica plates (see Fig. 1) and Δ, as given by equation (5), is the deviation from a quarter wave retardation:

$$\Delta = \frac{\pi}{2} \left\{ \frac{(n_1' - n_2')\lambda_1}{(n_1 - n_2)\lambda'} - 1 \right\}. \tag{5}$$

In this equation n_1' and n_2' are the indices of refraction of mica for wave length λ'.

Instead of Δ the total value of the retardation ψ' produced in the mica plates may be considered. In this case

$$f = \cos^2 2\beta + \sin^2 2\beta \sin^2 \psi' \tag{4a}$$

$$\psi' = d(n_1' - n_2') \frac{2\pi}{\lambda'} \tag{5a}$$

and from this

[2] *The Analysis of the Colors Observed in Photoelastic Experiments*, R. V. Baud and W. D. Wright, J.O.S.A., *20*, 7; 1930.

[3] Loc. cit., *The Analysis of the Colors Observed in Photoelastic Experiments*, p. 386.

$$\Delta = \psi' - \pi/2 \tag{6}$$

by definition.

DISCUSSION

In Fig. (2) curves are shown for five arbitrarily chosen values of Δ. These curves were so constructed that the vector OX drawn at the angle

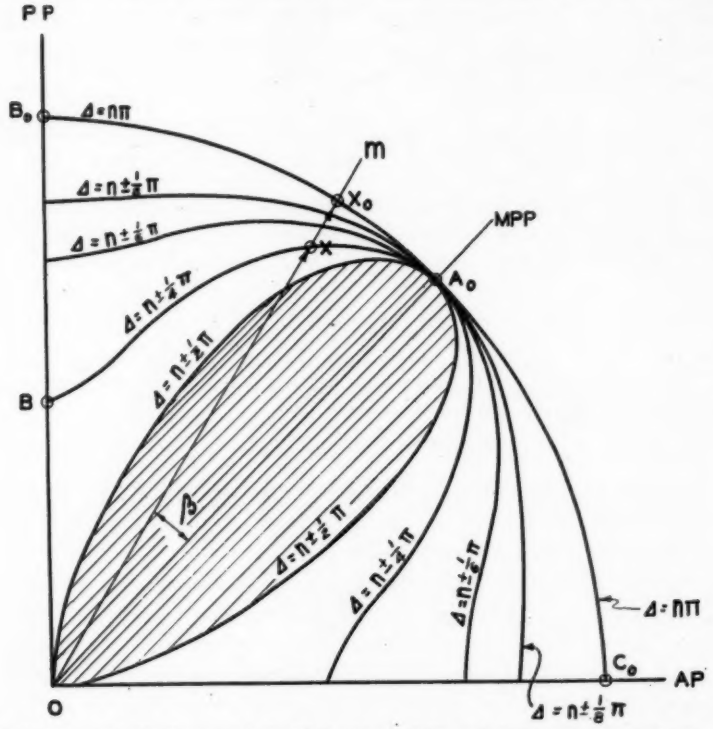

FIG. 2. *Diagram indicating magnitude of factor f for variable angle β and for five different values Δ.*

β to the axis MPP represents the magnitude of the factor f, equation (4), whereby the distance $OA_0 = OB_0 = OC_0$ represents unity. Whatever the combination of β and Δ is, the tip X of the vector OX will never be in the shaded area. A more important conclusion is that the reduction XX_0 from unity, see Fig. 2, is zero for $\beta = 0$ and a maximum BB_0 for $\beta = 45°$, furthermore that for $\beta = 22\ 1/2°$ the reduction XX_0 is half of the maximum reduction BB_0.

This seems to indicate, that in order to minimize the error that is

introduced due to taking a record of the isochromatics of the comparison strip in one arbitrary position only, instead of the positions that correspond with the axes in the model, the strip should be located at an angle of $\beta = 22\ 1/2°$. Further analytical and experimental work is necessary to evaluate the exact magnitude of the maximum error incurred, which we think should be well within 5 or 10% for a balanced equipment.

PHOTOMICROGRAPHY WITH THE 365mμ MERCURY ARC LINE

By A. P. H. Trivelli and L. V. Foster

[Joint Communication from the Research Laboratories of the Eastman Kodak Company, No. 456, and from the Scientific Bureau of Bausch and Lomb Optical Company, Rochester, N.Y. Received Oct. 27, 1930]

Introduction

In 1926 C. E. K. Mees[1] suggested that the 365mμ mercury arc line is particularly suited for photomicrography. The theoretical limit of resolving power with an objective of N.A. = 1.40 is for wave length

450 mμ:0.16μ

365 mμ:0.13μ

and with an objective of N.A. = 1.30 for wave length

450 mμ:0.17μ

365 mμ:0.14μ

where Köhler's ultraviolet photomicrographic outfit gives a maximum resolution of 0.11μ with an objective of N.A. = 1.25 using the 275mμ cadmium-line as light source.

This means that with an objective of N.A. = 1.40 photomicrography with the 365mμ mercury line is able, theoretically, to give 19 percent more resolving power than the best microscopic systems are able to give with the Wratten C filter. With an objective of N.A. = 1.30 the gain in resolving power would be 12 1/2 percent more. An exception is the objective of N.A. = 1.6, which, with λ450mμ, gives a resolution of 0.14μ, slightly less than could be obtained with an objective of N.A. = 1.4 using the 365 mμ wave length radiation. These objectives are, however, very rarely used for resolution work, but have their practical applicability in the measurement of the angles of the optical axes of biaxial crystals.

Without doubt the highest resolution is secured by Köhler's ultraviolet photomicrographic outfit, but micrography with λ365 mμ approaching this resolution has the following advantages over Köhler's apparatus:

(1) A glass system can be used instead of the expensive quartz system.

(2) The light source is a mercury arc lamp with filters, which is much

[1] Mees, C. E. K.: *Color Sensitivity of Photographic Materials*, J. Franklin Inst., *201*, 525; 1926.

simpler than the spectroscopic outfit used to obtain a high intensity cadmium line.

(3) The focusing can be greatly simplified, as will be explained.

In a previous investigation by R. P. Loveland and one of us[2] it was found that with a mercury arc of sufficient intensity the use of λ365 mμ is practical. In general, the absorption in glass is small enough and the available intensity of the 365 mμ mercury arc line sufficiently high to produce micrograms, in most cases with the highest magnifications within a few minutes exposure. The only difficulty was, however, that all commercial objectives were not corrected for λ365 mμ. This difficulty having been overcome by one of us, application of the 365 mμ wave length in photomicrography could be carried a step further.

THE LIGHT SOURCE AND THE OPTICAL SYSTEM

Köhler's ultraviolet photomicrographic outfit has no chromatic correction in the lenses and therefore requires a light source of a single spectroscopic line. According to J. M. Eder and E. Valenta's "Atlas of Spectra"[3] the arc spectrum of mercury in the near ultraviolet shows the following lines:

λ *in mμ*	*Relative Intensity*	λ *in mμ*	*Relative Intensity*
398.408	2	368.074	3
390.663	4	366.346	5
390.212	2	366.305	4
382.06	2	365.500	7
379.036	2	365.031	8
377.071	2	356.153	2
375.183	2	354.365	2
370.442	2	339.050	2

The lines most powerfully active on the photographic plate are the doublets of λ365 mμ and λ366 mμ. Wratten filter No. 18-A, which is used for the isolation of the 365 mμ mercury line, has its maximum transparency at 363 mμ, which lies very near the 365 mμ mercury line of which 38 percent is transmitted. The other weak lines are so much absorbed that, for instance, the action of the 390 mμ mercury line can be entirely ignored.

By increasing the intensity of the mercury arc by increasing the cur-

[2] Trivelli, A. P. H., and Loveland, R. P.: *Photomicrography of Silver Halide Grains with Ultraviolet Radiation*, J.O.S.A., *20*, 97; 1930.

[3] Eder, J. M., and Valenta, E.: *Atlas Typischer Spektren*, Ed. 2, Vienna, Akademie der Wissenschaften, 1928.

rent, the light becomes blinding white and the strong lines become broader. It is evident that the chromatic correction required by such a light source can be satisfactorily attained by a glass system.

In our investigations we used a quartz mercury vapor lamp of the atmospheric pressure type (Kelvin, Bottomley & Baird Ltd., S. R., No. W. D. 101; Type VA) with the special micro-outfit working on a 220 volt dc, with a starting current of 4.4 amperes, a running current of 2.2 amperes and a series resistance of 42 ohms. The condenser was replaced by a nonspherical aplanat with an aperture of f/1. This lamp gives a very intense and constant light. On the *M* plate an exposure of 15 to 30 seconds is sufficient with a 16 mm objective, a x5 hyperplane eyepiece and the photographic plate at 50 cm distance from the eyepiece.

An improvement over this lamp, however, suggested itself to R. W. Wood. He improved the intensity of the mercury arc by a cooling system employing running water.[4] It is quite possible, that in this way mercury arc lamps of higher intensity than are now available may be constructed.

We worked out two special objectives, one of 16 mm focal length and the other an oil immersion objective of 1.7 mm focal length which can be focused definitely for λ365 mμ. These objectives are designed to bring together the wave lengths 546 and 365 mμ, that is, achromatism has been accomplished for one wave length in the visible and for one wave length in the invisible spectrum. Thus, it is possible to examine the preparation visually by using the mercury green filter No. 62, or No. 77. After the focus has been determined, the 365 mμ filter is substituted and the exposure made. The result is a perfectly sharp picture of the object which reveals detail invisible to the eye.

The green mercury monochromat No. 62 transmits about 12 percent of the green line and 0.05 percent of the yellow lines. Filter No. 77 transmits about 72 percent of the green line and about 1/2 percent of the yellow lines, an amount negligible for most purposes. If the red lines of the quartz lamp are objectionable and must be eliminated, filter No. 58 should be superimposed on filter No. 77.

IMMERSION OIL

During the course of experiments in this work, it was found that cedar oil possessed a high absorption for λ365 mμ. Other oils were investigated for use as immersion media and the best oil for transmitting

[4] Wood, R. W.: *Raman Effect by Helium Excitation*, Phil. Mag., 7, 858; 1929; Pringsheim, P.: *Raman Spectra*. Handbuch der Physik, *21* (Licht und Materie); 609; 1929.

this wave length and having the proper optical properties was found to be sandalwood oil. The spectral transmission of this oil and of cedar oil is given in Fig. 1.

These curves show the absorption of cedar and sandalwood oil at a thickness of 0.2 mm. The curves are plotted from measurements made with oils mounted on a quartz slide 2.3 mm thick covered with a cover glass 0.2 mm thick. The absorption values include the absorption by

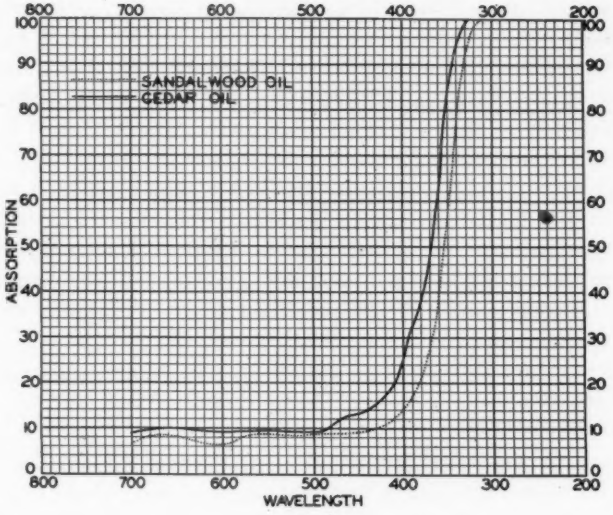

FIG. 1

the quartz and cover glass and reflections at the two surfaces. It can be seen from the curves that cedar oil has an absorption of 56 percent for λ365 mμ and sandalwood oil an absorption of 32 percent. These values are for a 0.2 mm layer of immersion oil, which is approximately the thickness between the cover glass of the mount and the objective. Since the requirements for full resolution of objects are that the condenser aperture must equal the aperture of the objective, the condenser must have immersion contact with the slide. This contact may require a layer of oil from 0.2 mm upward. In case the total thickness of oil used between the condenser and the slide and between the cover glass and the objective is as great as 1.0 mm the absorption of the cedar oil at λ365 mμ would be 98.4 percent, and for the same thickness the absorption of the sandalwood oil would be 85 percent. The transmission for cedar oil

would be 1.6 percent and for sandalwood oil 14.4 percent; that is, the latter would be nearly ten times as transparent as cedar oil. It can be easily seen that whichever oil is used it is well to keep the thickness of the immersion oil layer between the condenser and slide at a minimum.

THE MICROSCOPIC OBJECT

Most objects are mounted in Canada balsam, and only in special cases is realgar or another medium used. Several of these substances are opaque for λ365 mμ. A convenient and quick method by which to determine roughly the absorption of the balsam is by the use of an x-ray screen of barium platinocyanide, which when exposed to λ365 mμ radiation, gives a bright green fluorescence. In the beam of the ultraviolet radiation the slide forms a shadow on the x-ray screen, if the balsam absorbs the radiation.

The great advantage of the use of slides having Canada balsam as mountant lies in the fact that it is already used to a great extent in microscopic research work necessitating fixing and staining. These slides are useless for λ275 mμ photomicrography, but very useful for λ365 mμ photomicrography. The preparation of slides for λ275 mμ photomicrography is entirely in an experimental state.

Concerning the absorption, in the ultraviolet, of the stains used in these slides, very little is known. The National Aniline and Chemical Co., Inc., New York, at our request put a number of their stains at our disposal for the determination of their absorption spectra. All these stains were found satisfactory by the Commission on Standardization of Biological Stains. At our request, Mr. E. E. Richardson* determined for us the absorption for λ365 mμ and λ275 mμ in 1 cm layer of solution. This result is not to be regarded as the same when the stain is precipitated on the object, but it gives an approximate indication of what may be expected when the stain is used for ultraviolet micrography. The data are given in Table 1 in which the absorption is expressed in densities (D).

PHOTOGRAPHIC MATERIAL

For the great majority of cases in photomicrography the use of the Wratten M plate is recommended. It is advisable in practice to employ, as much as possible, the same photographic material in order to reduce the number of variables to a minimum. A faster plate is preferable only with dark field illumination, very oblique illumination, and

* Physics Dept., Research Laboratories, Eastman Kodak Company.

with polarized light. For photomicrography in the visible light the new Wratten hypersensitive panchromatic plate enables the shortest

TABLE I. *Ultraviolet absorption of stains*

Stain	Purity	Solvent	Dilution	Absorption for 365 mμ D	Absorption for 275 mμ D
*Acid fuchsine	62%	H_2O	1:7500	0.17	1.08
Aniline blue		"	1:5000	1.13	∼
Azure I		"	1:50,000	0.32	1.86
*Basic fuchsine	82%	"	1:20,000	0.14	0.70
Bismarck brown	43%	"	1:20,000	0.78	0.94
Brilliant cresyl blue	68%	"	1:25,000	0.60	1.54
Brilliant green	94%	"	1:40,000	0.18	0.48
Congo red	89%	"	Fades rapidly		
*Crystal violet	91%	"	1:50,000	0.12	0.51
Cresyl violet		"	1:20,000	0.40	1.66
Eosine B	91%	"	1:20,000	0.47	1.47
*Eosine Y	83%	"	1:20,000	0.32	0.98
Indigo carmine	89%	"	1:10,000	1.00	∼
Janus green	62%	"	1:20,000	0.88	1.86
Light green S. F.		"	1:20,000	0.20	0.87
Malachite green	78%	"	1:40,000	0.23	0.60
Martin's yellow		"	1:20,000	1.02	1.64
*Methyl green	56%	"	1:20,000	0.26	0.94
*Methyl violet	85%	"	1:50,000	0.25	0.67
*Methylene blue	88%	"	1:50,000	0.26	1.34
Methyl orange	97%	"	1:20,000	0.68	1.52
Neutral red		"	1:20,000	0.46	∼
Nile blue		"	1:30,000	0.46	2.23
Nigrosine		"	1:20,000	1.80	∼
Orange II	95%	"	1:20,000	0.67	1.40
*Orange G	83%	"	1:20,000	0.70	1.18
Phloxine		"	1:20,000	0.38	1.22
Pyronine		"	1:30,000	0.40	1.14
*Safranine O	94%	"	1:30,000	0.24	∼
Toluidine blue	83%	"	1:30,000	0.31	2.28
Thionine		"	1:40,000	0.29	2.20
*Carmine °)		$H_2O+Na_2CO_3$	1:5000	2.20	∼
Ethyl eosine		$H_2O+C_2H_5.OH$	1:20,000	0.39	1.38
Hematoxyline	C.P.	" "	1:10,000	2.72	2.54
Methylene violet		" "	1:10,000	0.35	2.38
*Sudan III	87%	" "	1:10,000	1.34	1.50
*Sudan IV	90%	" "	1:10,000	0.64	0.84

* The most commonly used stains.

°) Sodium carbonate has been used in 10% solution. The absorption for λ365 mμ is $D=0.14$, and for λ275 mμ $D=0.34$ in a 1 cm solution.

possible exposures to be given. In the visible light this plate has a higher speed than the Eastman Speedway Plate. In 365 mμ radiation, however, the Speedway plate is about three times faster than the Wratten hypersensitive panchromatic plate in the same radiation and should

FIG. 2

be used for 365 mμ photomicrography in extreme cases of low intensity, mentioned above.

DISCUSSION OF THE RESULTS

Photomicrography with radiation of λ365 mμ makes available a considerable increase of resolving power, or alternatively, if the resolving power obtained with visible light is sufficient, the use of the ultraviolet enables an increase of depth to be obtained by reduction of the aperture. Perhaps the greatest advantage in comparison with visible light is in the increased selective absorption shown by different proteins in histological sections. The value of this selective absorption is well known in connection with photomicrography by means of λ275 mμ, but the absorption which the proteins show for that wave length differs from that obtained when using λ365 mμ. For this reason the use of λ365 mμ is valuable even if equal resolving power were obtainable by the use of an objective of extremely high numerical aperture made for use with

visible radiation, and at the same time photomicrography with λ365 mμ cannot, in all cases, replace photomicrography with the Köhler equipment and λ275 mμ.

As an example of the difference in absorption, the photomicrograms

FIG. 3.

in Figs. 2 and 3 show a part of a Discodrellid worm. Figure 2 is taken with the green mercury line, and Fig. 3 with λ365 mμ. These figures show clearly the differences in absorption for these wave lengths displayed by the specimen.

THE PHOTOMETRIC SENSIBILITY OF THE EYE AND THE PRECISION OF PHOTOMETRIC OBSERVATIONS

By E. M. Lowry

[Communication No. 454 from the Kodak Research Laboratories, Rochester, New York. Received Nov. 11, 1930]

In this paper data are presented to show that for maximum photometric sensibility the brightness of the photometric field should be adjusted so that a brightness level of from 20 to 30 millilamberts is secured. At this brightness the difference fraction $\Delta B/B$ has a value of 1.37 percent. Further the precision of photometric observations as expressed by the average percentage deviation, from the mean, of a single observation is 0.19 percent at the above mentioned optimum brightness level. At this same field brightness the maximum percentage deviation from the mean of a single observation was found to be 0.41 percent.

The brightness level at which the normal human eye possesses maximum sensibility for brightness difference has, since the pioneer work of König and Brodhun, been the object of investigation by a number of observers. Interest in the question of brightness contrast arises from the fact that the ability of the eye to detect detail in an object or scene depends in a large measure upon the facility with which minute differences in brightness are perceived. The present investigation was undertaken primarily from the standpoint of the photometrician and the work was done on a photometer bench under necessarily highly specialized conditions. For example, the area of the retina stimulated was confined to the region immediately surrounding the fovea, that is, the position of maximum retinal sensibility.

König's data, as reduced by Nutting,[1] showed that maximum sensibility is reached at approximately 10 millilamberts. Nutting's, and Blanchard's[2] later experiments, yielded the same result. Holladay[3] found by a different method that the maximum sensibility to contrast appeared at about 10 to 25 ml. Uppenborn[4] reported that for maximum precision in photometric work a field illumination of 30 meter candles was required, while Dauber's[5] investigation yielded a value of from 125 to 250 meter candles.* It may be readily seen from the references given above that there is little agreement with regard to the effect of field brightness on photometric sensibility and the precision of observations.

The apparatus used for the work reported here consisted of the usual photometric equipment such as a three meter photometer bench calibrated in millimeters; a photometer head of ordinary design equipped with an eyeshade; several calibrated lamps having efficiencies of 17.6

* The data given by Uppenborn and Dauber were not sufficient to permit the reduction of the field illumination to field brightness in millilamberts.

lumens per watt and 19.9 lumens per watt and a potentiometer control for the lamps. The photometric cube was of the simple equality type with vertical dividing edge. A five millimeter exit pupil was used in the observation telescope and the field as seen through the instrument was so diaphragmed that it subtended approximately 3° at the eye of the observer.

TABLE 1. *Relation between field brightness and photometric sensibility of the eye.*

Field brightness millilamberts	Log B	$\Delta B/B$	Mean ad of single setting	Average maximum deviation of single setting
0.62	1.79	2.70%	0.60%	0.97%
2.24	0.35	2.09	.36	.60
6.61	0.82	1.69	.25	.48
13.8	1.14	1.50	.22	.43
25.1	1.40	1.37	.19	.41
55.0	1.74	1.37	.19	.42
97.7	1.99	1.38	.19	.43
145.0	2.16	1.42	.19	.47
224.0	2.35	1.45	.20	.49

In making observations a pair of calibrated lamps having the same rating in lumens per watt were placed in the lamp holders at either end of the bench, and one of the lamps and the photometer head were clamped in predetermined positions. The observer's eye was adapted to the brightness of the field by making ten settings of the adjustable lamp for equality of brightness in the field of view. The next step was to so move the lamp that one-half of the photometric field was just perceptibly brighter than the other, and the lamp position was recorded. A second adjustment was then made so that the previously light side now became the darker of the two. This process was repeated with the two halves of the field alternately bright and dark until ten settings had been made for each condition. Both lamps were then moved to positions nearer the photometer head in order to increase the brightness level and the procedure was repeated. In this way readings were secured at intervals for a range of brightness from 0.6 to approximately 250 ml. All brightness values were corrected for losses in the photometer head and represent the actual field brightness presented to the eye.

Six persons, each of whom had had considerable experience in photometric work, acted as observers, and each made a complete set of readings on three different days. In this way eighteen separate determinations were made at each of nine field brightnesses. Since ten settings

were made at each brightness, every point in the results below is the average of 180 separate readings.

RESULTS

Photometric sensibility is commonly expressed as the ratio of $\Delta B/B$, where ΔB is the least perceptible difference in brightness between the

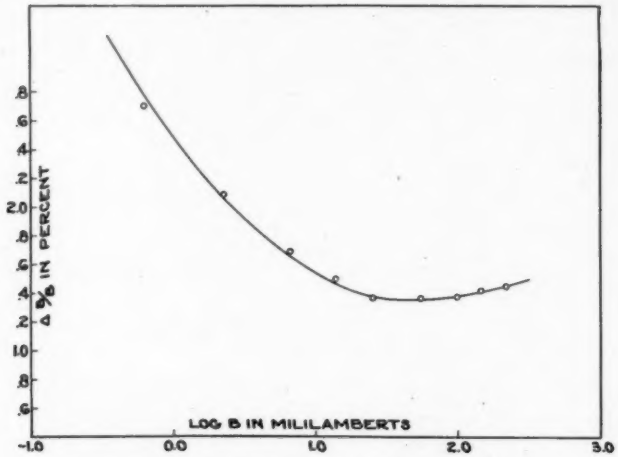

FIG. 1. *Relation field brightness and photometric sensibility.*

two halves of a photometric field and B is the field brightness. The average results for the eighteen sets of observations are given in Table 1. In Fig. 1 the ratio $\Delta B/B$ has been plotted against the field brightness B and it will be seen that this ratio reaches a minimum value of 1.37 percent at about 25 to 30 ml. Between 25 and 200 ml the value of the difference fraction $(\Delta B/B)$ is practically constant.

Since a minimum value for $\Delta B/B$ indicates maximum photometric sensibility, the data show that the eye attains its highest sensibility for contrast in a photometric field at a brightness of the order of 25 ml. This result is in substantial agreement with that of Holladay[3] mentioned above.

Although this series of experiments was originally planned only as an investigation of photometric sensibility, the lack of published data on the relation between field brightness and precision of observations made it seem desirable to examine the data further. In order to determine this

relation the settings for equality made during the adaptation period were used. The average percentage deviation, from the mean, of a single observation was calculated for each set of observations at each brightness level. Since the brightness values were not the same for every set, the mean of all the average deviations was found graphically and these values are given in the fourth column of Table 1, and are shown plotted in curve *A*, Fig. 2. As might be expected the curve has a

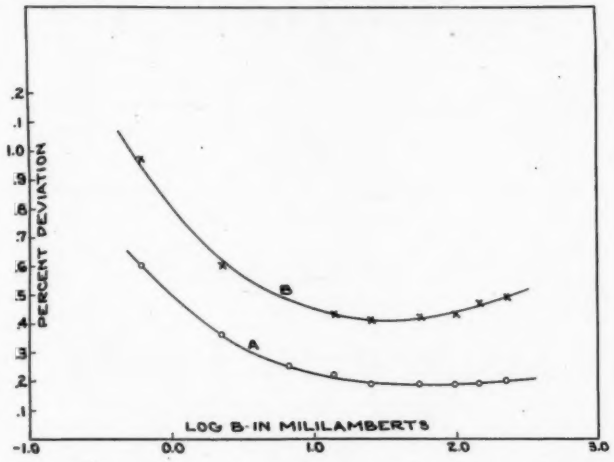

FIG. 2. *Relation between field brightness and precision of photometric observations. Curve A —average ad of single observation. Curve B—average maximum deviation of single observation.*

shape similar to that of the one for photometric sensibility. The highest precision as expressed by the lowest value of the average deviation of a single observation was 0.19 percent at about 25 ml.

In the same way the maximum percentage deviation, from the mean, of a single observation was found to be 0.41 percent at the same field brightness, namely at 25 ml, Table 1, column 5, and Fig. 2, curve *B*. Wild[6] used this method of maximum deviation as a measure of the sensitivity of various types of photometers.

From the above results it may be concluded that where the highest photometric accuracy is required the field brightness should be so adjusted that a level of at least 20 to 30 ml is secured.

In conclusion I wish to express my thanks to Mr. Loyd A. Jones for his interest and advice and to those men who so willingly acted as observers.

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